IOQA 2021-22 PART 1 (NSEA 2021-22)

SOLUTIONS OF PAPER CODE-41

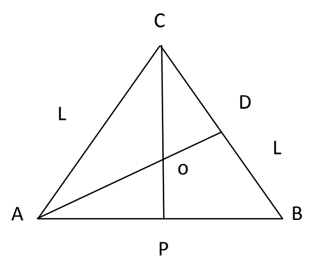
1. Due to inelastic collision, only momentum in conserved.
Balancing vertical component (y direction)of momentum before and after the collision:
$m_1 v_y + m_2 v_y = (m_1 + m_2) v_0^{final}$ (Note: both masses have same v_y).
So $v_y{}^{final} = v_y \Rightarrow$ Fused body(formed by the joining of the two masses) with mass $(m_1 + m_2)$ has identical velocity as any one of the masses had just before the collision(in y direction).Thus subsequent motion (vertical) of the fused body will be the same as that of any one of the masses.
Time taken inreaching the highest point by any one of the bodies is
$v_{y\ highest} = 0 = v_{0y} - gt = v\cos 45 - gt$
or $0 = \frac{v}{\sqrt{2}} - gt$ or $t = \frac{v}{g\sqrt{2}}$
Total time taken when this body falls to the ground will be twice of this
$= 2 \times \frac{v}{g\sqrt{2}} = \frac{v\sqrt{2}}{g}$
Thus, the total time after which the fused body falls to the ground is $= \frac{v\sqrt{2}}{g}$.

Ans : a

2. Let us assume $y = e^{-x} z$ then
$\Rightarrow y' = -e^{-x} z + z' e^{-x} = -y + z' e^{-x}$
or $y' + y = z' e^{-x} = \mathrm{A} e^{-x}$ (Given) $\Rightarrow \mathrm{z'} = \mathrm{A} \Rightarrow \mathrm{z} = \mathrm{Ax} + \mathrm{B}$
$\Rightarrow y = e^{-x}(\mathrm{Ax} + \mathrm{B}) \Rightarrow 5 = e^{-1}(\mathrm{A} + \mathrm{B}) \Rightarrow \mathrm{B} = 5\mathrm{e} - \mathrm{A}$
$y = e^{-x}(\mathrm{Ax} + 5\mathrm{e} - \mathrm{A}) = e^{-x}[(x-1)A + 5e]$
Ans : b

3. $m_c - m_s = 2.5\ log_{10} \frac{s_s}{s_c} \Rightarrow 1.6 - (-1.4) = 2.5\ log_{10} \frac{s_s}{s_c}$ or $log_{10} \frac{s_s}{s_c} = \frac{3}{2.5} = 1.2$
$\frac{s_s}{s_c} = 16 \Rightarrow S_s = 16 S_c$ where S is the flux density and we have used
$log_{10} 2 = 0.3 \Rightarrow 4\ log_{10} 2 = 1.2 \Rightarrow log_{10} 16 = 1.2$
Light gathering power or flux density $S \propto D^2$, where D being the telescope objective diameter.
To increase S by 16 $\Rightarrow$ we need $16S \propto 16D^2 = (4D)^2$ i.e. if D=1" then telescope for castor is 4D=4".
Ans : b

4. $L^2 - \frac{L^2}{4} = \frac{3L^2}{4}$ Therefore $AD = \frac{\sqrt{3}\ L}{2}$
Radius of the inscribing circle $\frac{2}{3} \times \frac{\sqrt{3}\ L}{2} = \frac{L}{\sqrt{3}}$
$Area\ of\ circle = \frac{\pi L^2}{3}$



Area of the triangle = $\frac{1}{2}\times L\times\frac{L\sqrt{3}}{2}=\frac{L^2\sqrt{3}}{4}$

Ratio $\frac{4\pi L^2}{3\times\sqrt{3}\ L^2}=\ \frac{4\pi}{3\sqrt{3}}$.

Ans : b

5. Frequency is a fundamental property of a wave and it does not alter when wave goes from one medium to other. Hence the frequency will not change when light goes from air to water, however both the speed and the wavelength decrease according to the relation
speed = frequency × wavelength.
Ans : a

6. If separation between the images of the two points is r_i then $\frac{r_i}{r_o}=\frac{v}{u}$ where v is the distance of the image from the lens. Due to diffraction each point will cast a diffuse image whose size is given by the angular width of the diffraction peak

$\theta = 1.22\frac{\lambda}{d}$ so the size of the diffraction spot for image of each point will be

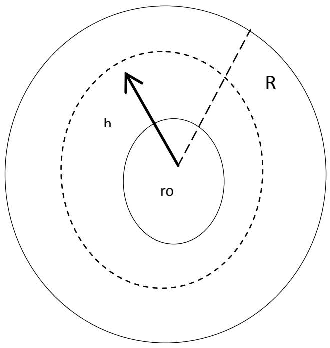
$$r_{diffr.} = v\theta = 1.22\frac{\lambda}{d}v$$

The image of the two points will be resolved it $r_{diffr.} < r_i$

$$\Rightarrow 1.22\frac{\lambda}{d}v < r_i = r_o\frac{v}{u} \quad \Rightarrow u < \frac{r_o d}{1.22\lambda}.$$

Ans : c

7. Mass contained within radius r is :



$$M_{(r)} = M_0 + \int_{r_o}^{r} 4\pi r^2 \rho(r) dr$$

Where M_o is the mass of the core of radius r_o

Now $\rho_{(r)} = \rho_o(\frac{r_o}{r})$ for $r_o \le r \le R$ so

$$M_{(r)} = M_o + 4\pi\rho_o r_o \int_{ro}^{r} \frac{r^2}{r} dr = M_o + 4\pi\rho_o r_o (\frac{r^2 - r_o^2}{2}) = M_o + 4\pi\rho_o r_o (\frac{r^2 - r_o^2}{2})$$

$= B' + A' r^2$ where $A' = \frac{4\pi\rho_o r_o}{2}$ and $B' = M_0 - \frac{4\pi\rho_o r_o^3}{2}$ Now gravitational acceleration at point r is entirely governed by $M_{(r)}$ with spherical symmetry, we have

$g_{(r)} = \frac{GM_{(r)}}{r^2} = A + \frac{B}{r^2}$ WhereA= A'G and B= B'G.

Ans : a

8. $f(1) = 5\ e^{-0} = 5 \qquad for\ x \le 1$

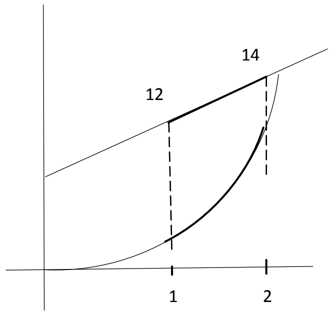
$f(1) = \frac{k}{25} = 5 \quad \Rightarrow \quad k = 125$

Ans : d

9. From the formula it follows, A D^2 = constant.
Therefore, $0.15\times 7^2 = A_{new}\times 10^2 \Rightarrow A_{new} = 0.07$
Ans : c

10. $\int_1^2 (2x+10)dx = \left(x^2+10x\right)\Big|_1^2 = 3+10 = 13$
$= \int_1^2 3x^2\,dx = x^3\Big|_1^2 = 8-1 = 7$

So the area between the line and the curve $13-7 =$ 6.
Ans :a



11. Summer triangle refers to three stars namely Vega, Altair and Deneb.
Ans : a

12. Given that A (1, 0, 5), B (2, 3, 1) and C (4, 9, r) Then
$\overrightarrow{AB} = -1i - 3j + 4k\ \ \overrightarrow{AC} = -3i - 9j + (5-r)k$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -1 & -3 & 4 \\ -3 & -9 & 5-r \end{vmatrix} = 0\ (must\,be)$$

$= i(-15+3r+36) - j(-5+r+12) + k(9-9)$
$= i(21+3r) - j(r+7) \Rightarrow 21+3r = 0$
or r = – 7 also r + 7 = 0 or r = – 7
Ans : c

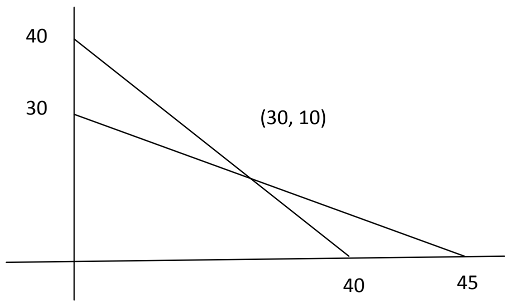
13. The distance in parsec is defined as :
$d\ in\ parsec = \dfrac{1}{p\ in\ \text{seconds of arc}}$ for small volume $d_s = \dfrac{1}{0.1} = 10\ parsec$ and for large volume $d_l = \dfrac{1}{0.025} = 40\ parsec$ Therefore, the volume of the larger sphere is $\left(\dfrac{40}{10}\right)^3 = 64\ times$ bigger. Thus the number of stars in large volume is $64\times 200\ = 12800$
Ans : c

14. According to the given constraint

$x+y \le 40 \Rightarrow x \le 40-y$ at least.

The other constraint is $2x+3y \le 90$ substituting

$80-2y+3y \le 90 \quad y=10, \; x=30$ just taking equal



Given that

$z=3x+4y+10$

$z = 120-3y+4y+10 = 130+y$

z increases with y on the x + y = 40 line, Now using $x=\frac{(90-3y)}{2}$

$z = 3\left(45-\frac{3}{2}y\right)+4y+10 = 145-\frac{y}{2}$

We check for the values at three points $(0, 30)(40, 0)(30, 10)$

$at(0,30) \;\; z=130$

$at(40,0) \;\; z=130$

$at(30,10)\, z=90+40+10 = 140$

z *is maximum at* $(30,10)$.

Ans : d

15. Force law remains that of a central Force. Hence, angular momentum is conserved. Hence for sure, the 2nd law remains correct. One cannot be sure about the other two laws (In fact they do change).

Ans : c

16. In the halo of our galaxy.

Ans : d

17. Knowing that $\prod_{n=1}^{\infty} \frac{1}{1-x^n} = \frac{1}{1-x} \times \frac{1}{1-x^2} \times \frac{1}{1-x^3} \times -----$

$= \left(1+x+(x)^2 ---\right)\left(1+x^2+(x^2)^2 ---\right)\left(1+x^3+(x^3)^2 ---\right)\left(1+x^4+---\right)\left(1+x^5+---\right)$

$= ---------+7x^5+-------$

x^5

$x^4 \cdot x$

$x^3 \cdot x^2$

$x^3 \cdot x \cdot x$

$x^2 \cdot x^2 \cdot x$

$x^2 \cdot x \cdot x \cdot x$

$x \cdot x \cdot x \cdot x \cdot x$

Ans : a

18. $a+b=6, \; ab = p, \; c+d=24, \; cd = q$

$a(1+r) = 6, \;\; a \cdot ar = p \;\; ar^2(1+r) = 24 \;\; \Rightarrow r=2, \;\; a=2$

$ar^2 \cdot ar^3 = q \;\; 2, \; 4, \; 8, \; 16$

$p=8=2^3 \;\; q=128=2^7 \quad \Rightarrow \quad pq=1024$

Ans :d

19. Given $V = \frac{1}{2} K X^2 \Rightarrow \frac{\Delta V}{V}\% = (\frac{\Delta K}{K} + 2\frac{\Delta X}{X})\%$

Given, $\frac{\Delta K}{K} = 0.5, \frac{\Delta X}{X} = 1$

Hence, $\frac{\Delta V}{V}\% = (0.5 + 2\times 1) = 2.5\ \%$

Ans : b

20. Let T be the orbital period of the faster and smaller orbit. Thus, $8T = 7.5\ days$ (since ½-day behind to start with). From Kepler's law $R^3 \propto T^2$,

$R_{gs}^3 = \lambda T_{gs}^2 \ and\ R^3 = \lambda T^2 = \lambda \left(\frac{7.5}{8.0}\right)^2$

Or $\frac{R}{R_{gs}} = \left(\frac{7.5}{8.0}\right)^{2/3} = \frac{3.84}{4} = 0.9578 \Rightarrow R = 0.96\ R_{gs}$

Therefore, R = 0.96 × 40000 ≈ 38400 km.

Ans :d

21. Height is obtained using the equation $v^2 = 0 = u^2 - 2\,gh$

where u and v are the vertical components of the initial and final velocity (at maximum height) respectively. Thus $h = \frac{u^2}{2g}$ we get: $\frac{h_{earth}}{h_{mars}} = \frac{g_{mars}}{g_{earth}}$ as u is assumed to be same for both cases.

Further knowing that $g = \frac{GM}{R^2}$, we get

$\frac{h_{earth}}{h_{mars}} = \left(\frac{M_{mars}}{M_{earth}}\right) \times \left(\frac{R_{earth}}{R_{mars}}\right)^2 = \left(\frac{1}{10}\right) \times 4 = \left(\frac{1}{2.5}\right) = 0.4 = \frac{2}{5}$

Ans : b

22. From the proportionality given, it follows that $\frac{R}{R_S} = \frac{\sqrt{L/L_S}}{(T/T_S)^2} \Rightarrow 20 = \frac{\sqrt{L/L_S}}{(1/2)^2} \Rightarrow \frac{L}{L_S} = 25$

Where R_s, T_s are the radius and surface temperature of the sun.From the HR- plot, the point (3000, 25) falls on Red giant.

Ans : d

23. **Ans: b**

24. **Ans: c**

25. Energy of the radiation emitted by the H atom in its rest frame is

$\Delta E = E_2 - E_1 = \left(\frac{-13.6}{2^2} - (-13.6)\right) eV \simeq 10.2\,eV$

As energy of the photon is $E = h\nu$, and the frequency of this photon will be blue shifted due to velocity of the H atom towards the 2nd H atom, the energy of this photon as received by the 2nd H atom will be

$\Delta E' = 10.2\ eV(1 + \frac{v}{c})$

In order to excite the electron from n=1 level to n=3, the energy required is

$$E_3 - E_1 = \left(\frac{-13.6}{3^2} - (-13.6) \right) \text{eV} \simeq 12.09\,\text{eV}.$$

$$Thus,\ we\ need\ \ 10.2(1+\frac{v}{c}) = 12.09\ or\ \frac{v}{c} = 0.185c\ = 5.55\times10^4\ km/\sec$$

Ans :a and d

26. **Ans : b and c**

27. Given that : $\sin\frac{\pi}{3} = 1 - 2\sin^2\theta = \cos 2\theta = \sin\left(\frac{\pi}{2} \pm 2\theta\right)$

$$\Rightarrow \frac{\pi}{3} = 2n\pi + \frac{\pi}{2} - 2\theta \text{ and } \frac{\pi}{3} = 2n\pi + \frac{\pi}{2} + 2\theta$$

$$\Rightarrow 2\theta = \frac{\pi}{6} + 2n\pi \Rightarrow \theta = n\pi + \frac{\pi}{12} \text{ and} \Rightarrow -2n\pi - \frac{\pi}{6} = 2\theta \Rightarrow \theta = -n\pi - \frac{\pi}{12}$$

$$\theta = n\pi - \frac{\pi}{12}.$$

Ans : a, b, c and d

28. $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$AA = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$ab + bd = 0 \Rightarrow b = 0\ or\ (a+d) = 0$$

$$ac + \text{dc} = 0 \Rightarrow \text{c} = 0\ or\ (a+d) = 0$$

If we take b =0 then from $bc + d^2 = 0 \quad \Rightarrow d = 0$

$$\Rightarrow a = 0\ or\ \ c = 0 \qquad \begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

If c =0 then a =0 too If c $\neq$ 0 then a=0

Otherwise we take , $a + d = 0 \Rightarrow d^2 + bc = 0 \Rightarrow bc - ad = 0$

$\det \text{A} = ad - bc = -d^2 - bc = 0 \Rightarrow det\ A = 0 \Rightarrow A^{-1}$ does not exist.

Ans : b and c

29. **Ans : b , c and d**

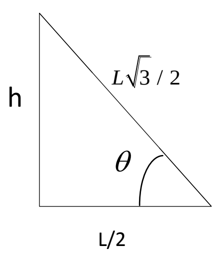
30. The slant height of the triangular face is

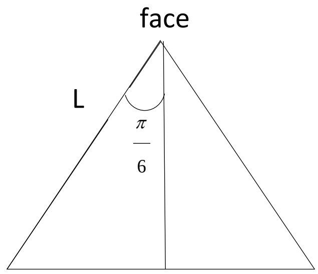
$$L\cos\frac{\pi}{6} = \frac{L\sqrt{3}}{2} \text{ and } \frac{3L^2}{4} = h^2 + \frac{L^2}{4}$$

$$h = \frac{L}{\sqrt{2}}$$

$$Area = L^2 + 4\times\frac{1}{2}\times\frac{\sqrt{3}}{2}L\times L = (1+\sqrt{3})\text{L}^2$$

$$\tan\theta = \frac{h}{L/2} = \frac{L/\sqrt{2}}{L/2} = \sqrt{2}$$





Volume of a cube is L^3 that of the pyramid is certainly less.

Ans : a , b and d

31. The emission line is towards longer wavelength, hence red – shifted compared to H_α, so the assertion (b) is wrong. NGC 1357 is a spiral galaxy, so relatively younger, and strong emission line close to H_α suggests high star building activities, so the option (d) is also wrong.

The red – shift z and subsequent velocity is $z = \frac{\lambda - \lambda_0}{\lambda} = \frac{6560 - 6606}{6560} \approx -0.007.$

Thereby $v = c \times z = 3 \times 10^5 \times 0.007 \approx 2.1 \times 10^3\ km/\sec$

From Hubble's law, $v = H_0 d \Rightarrow d = \frac{v}{H_0} = \frac{2.1 \times 10^3}{70} = 30 Mpc.$

Ans : a and c

32. A is contracting and B is expanding, so option (a) is wrong. Since A is contracting the galaxies in A are blue-shifted but in B they are red-shifted. The magnitude of velocity in A increases with distance, hence farther away they are greater is their recession velocity. Since, the velocity in A is higher than those closer, the red-shift z is larger for the farther galaxies.
Ans : b, c and d