INDIAN ASSOCIATION OF PHYSICS TEACHERS
NATIONAL STANDARD EXAMINATION IN PHYSICS
(NSEP – 2025)

Time: 120 minute **Max. Marks: 216**

Attempt All the Sixty Questions

A – 1

ONLY ONE OUT OF FOUR OPTIONS IS CORRECT. BUBBLE THE CORRECT OPTION.

Solution

1. The acceleration $a = -\frac{dv}{dt} \Rightarrow dv = -a\,dt \;\Rightarrow dv = -kv^2\,dt$.

   Integrating $\int \frac{dv}{v^2} = -k\int dt \;\Rightarrow -\frac{1}{v} = -kt - C \;\Rightarrow\; \frac{1}{v} = kt + C$

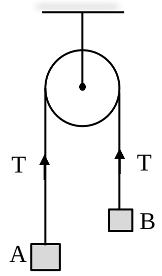
   At $t = 0,\;\; v = u \Rightarrow C = \frac{1}{u}$; therefore $\frac{1}{v} = kt + \frac{1}{u}$

   $\frac{1}{v} = \frac{1+kut}{u} \Rightarrow v = \frac{u}{1+kut}$ or $\frac{dx}{dt} = \frac{u}{1+kut} \Rightarrow dx = \frac{u}{1+kut}\,dt$ integrating we get

   $x = \frac{u}{ku}\ell n(1+kut) + C'$ Also at t = 0, x = 0 hence $C' = 0$, thus $x = \frac{1}{k}\ell n(1+kut)$

   Ans: a

2. See diagram ; let the mass of blocks A and B be m_1 and m_2 respectively
   Let $m_1 > m_2$ then one can find

   The acceleration $a = \frac{m_1 - m_2}{m_1 + m_2} g$ and tension $T = \frac{2m_1 m_2}{m_1 + m_2} g$

   $\Rightarrow\; m_1 - m_2 = \left(\frac{a}{g}\right)C$ and $2m_1 m_2 = \left(\frac{T}{g}\right)C$

   Since $m_1 + m_2$ = C constant (say) ; further g is also a constant

   Now using $(m_1 + m_2)^2 = (m_1 - m_2)^2 + 4m_1 m_2$

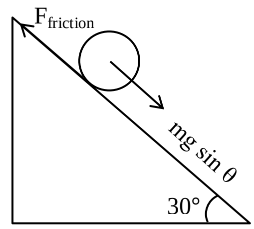
   Substituting $C^2 = \left(\frac{a}{g}\right)^2 C^2 + \frac{2T}{g}C \;\Rightarrow\; T = \left(\frac{gC}{2}\right) - \frac{C}{2g}a^2$

   or $T = \text{constant} - k a^2$ where $k = \frac{C}{2g}$

   $\therefore\; T = -k a^2 + \text{constant}$. This is the equation of straight line between T and a^2 with negative slope

   Ans: d

3. Acceleration of a rolling body down an incline is

   $a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}} = \frac{g\sin 30°}{2} = \frac{g}{4}$; for a rolling ring $K = R$

   Equation of motion for the rolling body is $mg\sin\theta - F_{friction} = ma$

   Substituting $a = \frac{g}{4}$ we get $mg\sin 30° - \frac{mg}{4} = F_{friction} \Rightarrow F_{friction} = \frac{mg}{4}$: b is correct

   Also $F_{friction} \le \mu mg\cos\theta \Rightarrow \frac{mg}{4} \le \mu mg\frac{\sqrt{3}}{2} \Rightarrow \mu \ge \frac{1}{2\sqrt{3}}$: c is wrong

   Ans: b

4. suppose the bullet is coming with velocity u, then its $KE = \frac{1}{2}mu^2 = FS$ = work done ….(1)

F being the opposing force inside the block. When the block is free to move, the conservation of momentum provides

$$mu = (m+M)V \Rightarrow V = \frac{mu}{m+M}$$

Hence the energy of the bullet & block system after the collision is

$$\frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\frac{m^2u^2}{(m+M)^2}$$

Therefore the energy retained by the bullet is $\frac{1}{2}mu^2 - \frac{1}{2}mu^2\frac{m}{(m+M)}$

$$= \frac{1}{2}mu^2\left[1 - \frac{m}{m+M}\right] = \frac{1}{2}mu^2\left[\frac{m+M-m}{m+M}\right] = \frac{1}{2}mu^2\left[\frac{M}{m+M}\right]$$

Now the bullet penetrating with this energy will go up to a distance S' inside the block

So $\frac{1}{2}mu^2\frac{M}{m+M} = FS'$ ….(2)

Dividing equation (2) by equation (1) $\dfrac{\frac{1}{2}mu^2\frac{M}{m+M}}{\frac{1}{2}mu^2} = \dfrac{FS'}{FS} \Rightarrow S' = \dfrac{M}{m+M}S$

Ans: d

5. The Q value of a nuclear reaction is calculated using Einstein's mass energy relation $E = mc^2$.

In case of the given nuclear reaction (β-decay), the energy released i.e. the Q value is

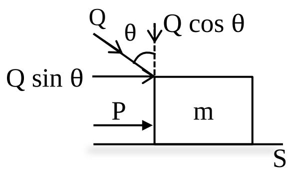
Q = [Mass of Al nucleus – (Mass of Mg nucleus + Mass of positron + mass of ν (neutrino))] c^2
Q = [(Mass of Al atom – 13 m_e) – (Mass of Mg atom – 12 m_e + positron mass)] c^2 or
Q = [24.990432 – 24.985839 – 2 m_e] × 931.5 MeV
Q = 0.004593 × 931.5 – 2 × 0.511 = 3.256 MeV

Ans: c

6. For preventing the block to slide

$$P + Q\sin\theta \le \mu(mg + Q\cos\theta)$$

Or $\mu \ge \dfrac{P + Q\sin\theta}{mg + Q\cos\theta}$



Ans: a

7. The decrease in potential energy when a mass m falls to Earth surface from height h

$$\Delta U = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = GMm\left(\frac{R+h-R}{R(R+h)}\right)$$

Now using $\frac{GMm}{R^2} = mg$ or $GMm = mgR^2$ we get $\Delta U = mgR^2\left(\dfrac{h}{R\left(R+\frac{R}{5}\right)}\right) = \frac{5}{6}mgh$

Ans: c

8. The acceleration $a = \sqrt{\left(\frac{v^2}{R}\right)^2 + 2^2} = \sqrt{\left(\frac{30^2}{600}\right)^2 + 2^2} = 2.5\,ms^{-2}$

Ans: b

9. The basic equations for projectile motion, with standard symbols, are

$x = u\cos\theta\, t \qquad y = u\sin\theta\, t - \frac{1}{2} g\, t^2$ and $T = \frac{2u\sin\theta}{g}$

At time t_1 : $y = h_1$ and at time t_2 : $y = h_2$ so

$$h_1 = u\sin\theta\, t_1 - \frac{1}{2} g\, t_1^2 \Rightarrow \frac{h_1}{t_1^2} = \frac{u\sin\theta}{t_1} - \frac{1}{2} g \quad \text{….. (1)}$$

$$h_2 = u\sin\theta\, t_2 - \frac{1}{2} g\, t_2^2 \Rightarrow \frac{h_2}{t_2^2} = \frac{u\sin\theta}{t_2} - \frac{1}{2} g \quad \text{….. (2)}$$

(1) – (2)

$$\frac{h_1}{t_1^2} - \frac{h_2}{t_2^2} = \frac{u\sin\theta}{t_1} - \frac{u\sin\theta}{t_2} \Rightarrow h_1 t_2^2 - h_2 t_1^2 = u\sin\theta\,(t_1\, t_2)(t_2 - t_1) \quad \text{…… (3)}$$

Also rewriting (1) and (2) as

$$h_1 = u\sin\theta\, t_1 - \frac{1}{2} g\, t_1^2 \Rightarrow \frac{h_1}{t_1} = u\sin\theta - \frac{1}{2} g\, t_1 \quad \text{….. (4)}$$

$$h_2 = u\sin\theta\, t_2 - \frac{1}{2} g\, t_2^2 \Rightarrow \frac{h_2}{t_2} = u\sin\theta - \frac{1}{2} g\, t_2 \quad \text{….. (5)}$$

(4) – (5)

$$\frac{h_1}{t_1} - \frac{h_2}{t_2} = \frac{1}{2} g\,(t_2 - t_1) \Rightarrow h_1 t_2 - h_2 t_1 = \frac{1}{2} g (t_1\, t_2)(t_2 - t_1) \quad \text{…. (6)}$$

Dividing equation (3) by equation (6)

$$\frac{h_1 t_2^2 - h_2 t_1^2}{h_1 t_2 - h_2 t_1} = \frac{u\sin\theta\,(t_1\, t_2)(t_2 - t_1)}{\frac{1}{2} g (t_1\, t_2)(t_2 - t_1)} = \frac{2u\sin\theta}{g} = T$$ Thus we can write

$$T = \frac{h_1 t_2^2 - h_2 t_1^2}{h_1 t_2 - h_2 t_1}$$

Ans: a

10. The normal reaction on the block of mass m is $N_1 = mg$

The force of friction between m and M is $f_k = \mu_k mg = \mu mg$

f_k acts against the direction of motion.

The applied force $F = 2\mu mg$ (given: acting rightward)

So m is accelerated w.r.t the plate and the equation of motion is

$2\mu mg - \mu_k mg = ma \Rightarrow 2\mu mg - \mu mg = ma \Rightarrow a = \mu g$

Equal and opposite force will act on upper surface of M.

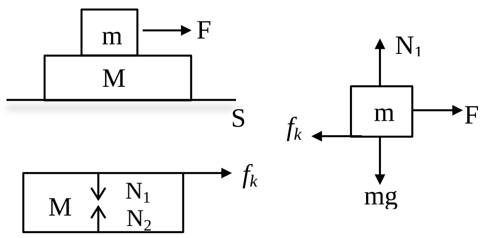
No friction or external force acts on M.

∴ from Newton's law

$f_k = MA$: A being the acceleration of M

$\mu mg = MA \quad \Rightarrow A = \frac{\mu mg}{M}$ in the direction of F

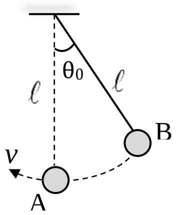
Ans: a

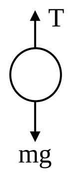


11. At the position B, the bob is instantaneously stationary. When it is released, its Potential Energy $mg\ell\left(1-\cos\theta_0\right)$ is converted in to KE. Therefore at A $\frac{1}{2}mv^2 = mg\ell\left(1-\cos\theta_0\right)$

$\Rightarrow v^2 = 2g\ell\left(1-\cos\theta_0\right)$

Thereby at A : $T - mg = \frac{mv^2}{l} \Rightarrow T = mg + \frac{m}{\ell}2g\ell\left(1-\cos\theta_0\right)$





$T = 3mg - 2mg\cos\theta_0$ Using T = 2 mg (given)

$2mg = 3mg - 2mg\cos\theta_0 \Rightarrow \cos\theta_0 = \frac{1}{2}$

$\therefore \ \theta_0 = 60°$

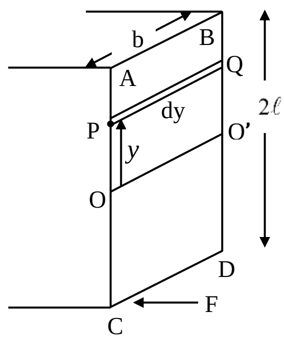
Ans: b

12. By symmetry the four planets can be held at diametrically opposite points means at the four corners of a square whose diagonal is the diameter of the circular path. Hence each side of the square is $a = r\sqrt{2}$. The resultant gravitational force on each planet is $F = \frac{Gmm}{(2r)^2} + \frac{2Gmm}{a^2} \times \cos 45°$. This serves as centripetal force hence $\frac{Gmm}{(2r)^2} + \frac{2Gmm}{a^2} \times \cos 45° = \frac{mv^2}{r}$

Or $\frac{Gm}{4r} + \frac{2Gm}{2r} \times \frac{1}{\sqrt{2}} = v^2$ or $v^2 = \frac{Gm}{r}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right]$ or $v = \sqrt{\frac{Gm}{r}\left[\frac{1+2\sqrt{2}}{4}\right]}$

Ans: a

13. AB is the top line of the door, OO' is the axis of rotation and CD is the bottom line of the vertical door of area $(2\ell \times b)$. Huge column of water stands against the door and exerts horizontal force on the door. The horizontal force caused by water pressure exerts a clockwise torque on the surface of the door above the axis OO' (hinge) and anticlockwise torque on the surface of the door below it.
The resultant torque tends to rotate the door about the hinge.
To calculate the resultant torque, let us consider a horizontal water layer PQ of width dy at a vertical distance y above the axis OO'.
Pressure at the layer PQ is $= \rho g\,(\ell - y)$
(pressure is due to water column above the considered layer of width dy)
Therefore the force against the layer PQ of area b dy on the door is
$dF = \rho g\,(\ell - y).b\,dy$
Moment of the force exerted by the water column at the layer PQ about the axis OO' is $d\tau = y \times dF = y \times \rho g\,(\ell - y).b\,dy = \rho g b\,(\ell - y)y\,dy$
Taking the clockwise moment positive and anticlockwise moment negative, the resulting torque is $\tau = \int_{-\ell}^{\ell} \rho g\, b\,(\ell - y)\,y\ dy = \rho g\, b\left[\int_{-\ell}^{+\ell} \ell y\,dy - \int_{-\ell}^{+\ell} y^2\,dy\right]$



$$\rho gb\left[\frac{\ell y^2}{2} - \frac{y^3}{3}\right]_{-\ell}^{+\ell} = \rho gb\left[\left(\frac{\ell^3}{2} - \frac{\ell^3}{2}\right) - \left(\frac{\ell^3}{3} + \frac{\ell^3}{3}\right)\right] = -\frac{2}{3}\rho gb\ell^3$$

Net torque is anti-clockwise. If a force F is to be applied at the bottom of the door to prevent it from rotating, the moment of this force must be equal and opposite to the resultant torque hence

$\ell \times F = \frac{2}{3}\rho gb\ell^3$ or $F = \frac{2}{3}\rho gb\ell^2$ Substituting $b = 2\,m$ and $\ell = 1\,m$, we get $F = \frac{4}{3}\rho g$

Ans: b

14. Using Poiseuille's formula, the rate (volume per second) of streamline flow of liquid, is

$Q = \frac{\pi P r^4}{8nl}$ or $P = \frac{Q \times 8nl}{\pi r^4}$ If pressure difference is h mm of mercury then

$$h \times 10^{-3} \times 13.6 \times 10^3 \times 9.8 = \frac{5 \times 10^{-6} \times 8 \times 4.0 \times 10^{-3} \times 1}{3.14 \times (0.4 \times 10^{-2})^4}$$

This gives h = 1.49 ≅ 1.5 mm

Ans: c

15. Pressure $[P] = ML^{-1}T^{-2}$, Velocity $[c] = LT^{-1}$, Energy per unit area per sec $[E] = \frac{ML^2T^{-2}}{L^2T} = MT^{-3}$

Therefore $P^xE^yc^z \equiv [ML^{-1}T^{-2}]^x\ [MT^{-3}]^y\ [LT^{-1}]^z = M^0\,L^0\,T^0$ = dimensionless

Or $P^xE^yc^z = M^{x+y}\,L^{-x+z}\,T^{-2x-3y-z} = M^0\,L^0\,T^0$ (given)

$\Rightarrow x + y = 0 : \quad -x + z = 0 : \quad -2x - 3y - z = 0$

$\Rightarrow y = -x \ : \ z = x$

$\therefore \ x : y : z \quad 1 : -1 : 1$

$\therefore \ x = \lambda, \ \ y = -\lambda, \ \ z = \lambda$ putting in third equation $-2x - 3y - z = 0$

Given $-2\lambda + 3\lambda - \lambda = 0$

Hence by taking $\lambda = 1$, we get $x = 1, \ \ y = -1, \ \ z = 1$

Ans: c

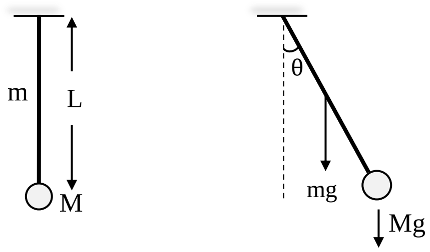
16. The volume of water coming out per sec is $a_1v_1 = a_2v_2 \Rightarrow a^2\sqrt{2gh} = \pi r^2\sqrt{2g \times 4h}$

$$\Rightarrow a^2 = 2\pi r^2 \Rightarrow r^2 = \frac{a^2}{2\pi}$$

Therefore $r = \frac{a}{\sqrt{2\pi}}$

Ans: c

17. Let the pendulum make an angle θ with the vertical.



The restoring torque $\tau = -mg\frac{L}{2}\sin\theta - MgL\sin\theta$

$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$ where $I = ML^2 + \frac{mL^2}{3}$

For small angle, sin θ = θ

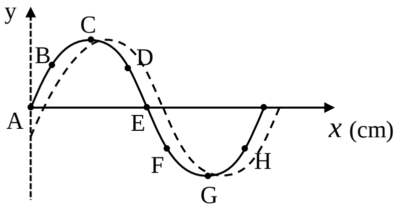
$$\left(ML^2 + \frac{mL^2}{3}\right)\frac{d^2\theta}{dt^2} = -\left(\frac{mL}{2} + ML\right)g\,\theta$$

$$\left(M + \frac{m}{3}\right)L\frac{d^2\theta}{dt^2} = -\left(\frac{m}{2} + M\right)g\,\theta \quad \text{or} \quad \frac{d^2\theta}{dt^2} = -\omega^2\,\theta \ \text{ where } \ \omega^2 = \frac{\left(M + \frac{m}{2}\right)}{\left(M + \frac{m}{3}\right)}\frac{g}{L}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\left(M + \frac{m}{3}\right)}{\left(M + \frac{m}{2}\right)}\frac{L}{g}} \quad \text{or} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{(m + 3M)}{(m + 2M)}\frac{2L}{3g}}$$

Ans: b

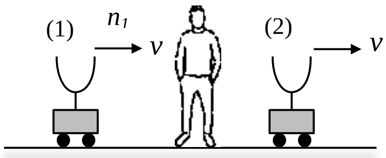
18. The continuous curve shows the wave at time t and the dashed curve at time t + Δt. The wave is travelling to the right. A going down ↓, B going down ↓, C is at top where the particle is at rest momentarily, D going up ↑, E going up ↑, F going upward, G is the lowest point where the particle is at rest momentarily, H going down ↓
So (d) is correct



Ans: d

19. The frequency of both the turning forks is n = 700; v_s = 350 ms^{-1}

Frequency n_1 heard from (1) is $n_1 = n\dfrac{v_s}{v_s - v}$



Frequency n_2 heard from (2) is $n_2 = n\dfrac{v_s}{v_s + v}$

Number of beats heard $= n_1 - n_2 = 4$ (given)

$$4 = n\frac{v_s}{v_s - v} - n\frac{v_s}{v_s + v} = nv_s\frac{2v}{v_s^{\,2} - v^2}$$

As number of beats $\Delta n << n$, $v << v_s$

∴ v^2 is negligible as compared to v_s^2

∴ $\Delta n = 4 = n\dfrac{2v}{v_s} = 700\dfrac{2v}{350}$ giving $v = 1\ ms^{-1}$

Ans: c

20. Average molar mass of the mixture is $M = \dfrac{n_1M_1 + n_2M_2}{n_1 + n_2} = \dfrac{1\times 4 + 2\times 32}{1+2} = \dfrac{68}{3}\,gm$

Using, for He, $C_P = \dfrac{5}{2}R$ and $C_V = \dfrac{3}{2}R$ also for O_2, $C_P = \dfrac{7}{2}R$ while $C_V = \dfrac{5}{2}R$

The C_P of the mixture $= \dfrac{1\times\frac{5}{2}R + 2\times\frac{7}{2}R}{1+2} = \dfrac{19}{6}R$ and

the C_V of the mixture $= \dfrac{1\times\frac{3}{2}R + 2\times\frac{5}{2}R}{1+2} = \dfrac{13}{6}R$

∴ $\gamma_{mixture} = \dfrac{C_P}{C_V} = \dfrac{19}{13}$

Speed of the sound in the mixture $v = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M}}$ where ρ = density

$$v = \sqrt{\frac{\frac{19}{13}\times 8.31\times 300}{\frac{68}{3}\times 10^{-3}}} = \sqrt{\frac{19}{13}\times\frac{8.31\times 300\times 3}{68\times 10^{-3}}} = 401\,ms^{-1}$$

Ans: d

21. All the three plates are very close and are perfectly black bodies

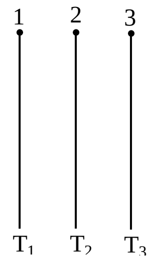
Radiation incident on the plate (2) from (1) $= \sigma . A T_1^4$ where $T_1 = 3T$ …. (1)

Radiation incident on the plate (2) from (3) $= \sigma . A T_3^4$ where $T_3 = 2T$ …. (2)

The total radiation absorbed by the plate (2) $= \sigma . A T_1^4 + \sigma . A T_3^4$

If temperature of the plate (2) is T_2 the radiation emitted by plate (2) $= \sigma . 2 A T_2^4$

In equilibrium $\sigma . 2 A T_2^4 = \sigma . A T_1^4 + \sigma . A T_3^4$



$2T_2^4 = (3T)^4 + (2T)^4$ from equations (1) and (2), we get

$T_2^4 = \frac{97}{2} T^4$ or $T_2 = \left(\frac{97}{2}\right)^{\frac{1}{4}} T$

Ans: c

22. Let σ be the mass per unit area of the disc.

Mass of cavity cut out from the disc is $M_2 = \pi b^2 \sigma$ with center $\vec{r}_2 = c\,\hat{i}$

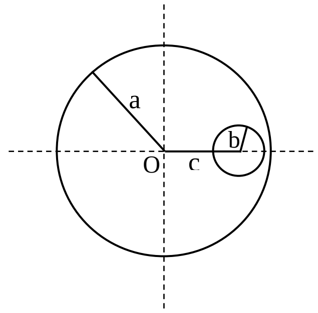
The mass M_1 left after the cavity is made is

$M_1 = \pi a^2 \sigma - \pi b^2 \sigma = \pi (a^2 - b^2) \sigma$

Let $\vec{r}$ be the center of mass of the remaining mass M_1.

The center of mass of complete disc (with no cavity) is at point O

and the mass of the complete disc is $M = M_1 + M_2 = \pi a^2 \sigma$



Using $\vec{r}_{CM} = \frac{M_1 \vec{r}_1 + M_2 \vec{r}_2}{(M_1 + M_2)}$ we get

$$0 = \frac{\pi\sigma (a^2 - b^2)\vec{r} + \pi\sigma b^2 (c\,\hat{i})}{\pi\sigma a^2}$$

$$\therefore \ \vec{r} = -\frac{b^2 c}{(a^2 - b^2)}\ \hat{i} \Rightarrow \left(-\frac{b^2 c}{(a^2 - b^2)}, 0\right)$$

Ans: d

23. Energy given to the gas is $\Delta Q = 8.31 \text{watt-hour} = 8.31 \times 3600\,J$

$$\Delta Q = n C_P \Delta T = 1 \times \frac{5}{2} R\,\Delta T = \frac{5}{2} \times 8.31\ \Delta T \quad \therefore\ \frac{5}{2}\Delta T = 3600\,J \Rightarrow \Delta T = \frac{2 \times 3600}{5} = 1440\,K$$

$T_2 - T_1 = 1440$ or $T_2 = (27 + 273) + 1440 = 1740$ K not 1740 °C ; a is wrong

$\Delta W = nR\Delta T = 1 \times 8.31 \times 1440 = 1440R$; b is wrong

$\Delta Q = 3600\,R \ \ \therefore \Delta U = \Delta Q - \Delta W = 3600R - 1440R = 2160R$; c is wrong

At constant pressure $\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{1740}{300} = 5.8$

Ans: d

24. The potential at any point inside the solid sphere of radius R with uniformly distributed charge q, is

$V = \frac{1}{4\pi \in_0} \frac{q}{2R^3}\left(3R^2 - r^2\right)$ where r is the distance of the point from the center

Thereby the potential at the center of the sphere is $V = \frac{1}{4\pi \in_0} \frac{3q}{2R} = V_1$ (say)

$$V_1 = \frac{3}{2} \frac{1}{4\pi \in_0} \frac{q}{R} = \frac{3}{2} \frac{1}{4\pi \in_0} \frac{4\pi R^3 \rho}{3R} = \frac{\rho R^2}{2 \in_0} = \frac{6\rho R^2}{12 \in_0} \quad \ldots\ldots (1)$$

The potential at the center A of the sphere due to the charge inside the cavity region with the same charge density ρ is $V_2 = \frac{1}{4\pi \in_0} \frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho}{\frac{R}{2}}$ or $V_2 = \frac{\rho R^2}{12 \in_0}$

Hence when the cavity is there, the net potential at A is

$V = V_1 - V_2 = \frac{6\rho R^2}{12 \in_0} - \frac{\rho R^2}{12 \in_0} = \frac{5\rho R^2}{12 \in_0} = \frac{k\rho R^2}{12 \in_0}$ thus k = 5

Ans: b

25. The energy flowing per sec per unit area of cross section in an electromagnetic wave can be written as

$$S = \frac{E \times B}{\mu_0} = 1400 \ \frac{J}{\sec m^2} \text{ (given)}$$

Also we know that $\frac{E}{B} = c$ (speed of light in vacuum) therefore using $B = \frac{E}{c}$

$$S = \frac{E^2}{c\mu_0} = 1400 \Rightarrow E^2 = 3 \times 10^8 \times 4\pi \times 10^{-7} \times 1400$$

$$E = \sqrt{12 \times 1.4\pi} \times 10^2 = 7.265 \times 10^2 = 726.5 \ Vm^{-1}$$

$$B = \frac{E}{c} = \frac{726.5}{3 \times 10^8} = 2.42 \times 10^{-6} tesla$$

$$B = 2.42 \times 10^{-6} T = 2.42 \ \mu T$$

Ans: a

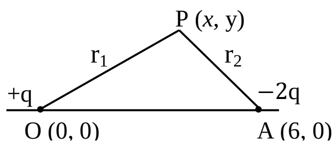
26. Looking at the output C and the inputs A and B, we observe that C is given by the truth table

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

This is table of AND gate

Ans: a

27. Potential at P is V = $V_1 + V_2 = 0$ (given)



or $\frac{q}{\sqrt{x^2 + y^2}} + \frac{-2q}{\sqrt{(x-6)^2 + y^2}} = 0$

or $(x-6)^2 + y^2 = 4(x^2 + y^2)$

$x^2 - 12x + 36 + y^2 = 4x^2 + 4y^2$

or $3x^2 + 3y^2 + 12x = 36$

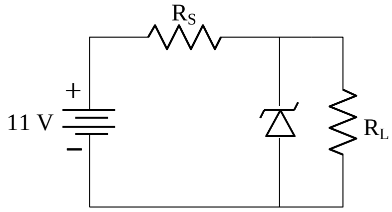
$x^2 + y^2 + 4x = 12 \Rightarrow x^2 + 4x + 4 + y^2 = 12 + 4$

$(x+2)^2 + y^2 = 4^2$

It is a circle of radius 4 and center (− 2, 0)

Ans: c

28. According to Kirchhoff's law,
current through the series combination shall be
$i = \dfrac{11}{(10+1)\times 10^3} A = 1 mA$ through each of the R_S and R_L



Further since the voltage across the load R_L is
$V_L = 10^{-3} \times 1 \times 10^3$ V = 1 volt,
less than the breakdown voltage hence no current through the zener.

Ans: b

29. Average current $= \dfrac{\Delta q}{\Delta t} = \dfrac{100\times10^{-3}\times200\times10^{-9}}{5\times10^{-3}} = 4\times10^{-6} = 4\,\mu A$; a is wrong

Peak current is = 100 mA for duration of 200 ns hence the charge is $= 100\times10^{-3}\times200\times10^{-9}$ which brings $= \dfrac{100\times10^{-3}\times200\times10^{-9}}{1.6\times10^{-19}}$ electrons

so the peak energy is $= \dfrac{100\times10^{-3}\times200\times10^{-9}}{1.6\times10^{-19}}\times50\times10^{6}\times1.6\times10^{-19} = 1$ J

hence the peak power is $= \dfrac{1}{200\times10^{-9}} = 5.0\ MW$; b is wrong

Also the average power delivered by the electron beam will be $= \dfrac{1}{5\times10^{-3}} = 200\ W$; c is correct

Ans: c

30. Magnetic field B_1 at O due to current in wire P is $B_1 = \dfrac{\mu_0 I}{2\pi r_1} = \dfrac{\mu_0 I}{2\pi 4}$

perpendicular to PO as shown in figure (1).

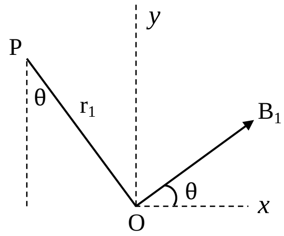


Figure (1)

Resolving B_1 along x and y axes $\vec{B}_1 = B_1\cos\theta\,\hat{i} + B_1\sin\theta\,\hat{j}$

Magnetic field B_2 due to current in wire Q is $B_2 = \dfrac{\mu_0 I}{2\pi r_2} = \dfrac{\mu_0 I}{2\pi\times3}$

perpendicular to QO as shown in figure (2).

Resolving B_2 along x and y axes $\vec{B}_2 = B_2\sin\theta\,\hat{i} - B_2\cos\theta\,\hat{j}$

adding components along x and y axes, the resultant B is

$\therefore\ \vec{B} = \vec{B}_1 + \vec{B}_2 = \dfrac{\mu_0 I}{2\pi4}\left(\dfrac{4}{5}\hat{i} + \dfrac{3}{5}\hat{j}\right) + \dfrac{\mu_0 I}{2\pi3}\left(\dfrac{3}{5}\hat{i} - \dfrac{4}{5}\hat{j}\right)$

$\therefore\ \vec{B} = \dfrac{\mu_0 I}{10\pi}(1+1)\hat{i} + \dfrac{\mu_0 I}{10\pi}\left(\dfrac{3}{4} - \dfrac{4}{3}\right)\hat{j}$

$\therefore\ \vec{B} = \dfrac{\mu_0 I}{10\pi}\left(2\hat{i} - \dfrac{7}{12}\hat{j}\right) = \dfrac{\mu_0 I}{5\pi}\left(\hat{i} - \dfrac{7}{24}\hat{j}\right)$

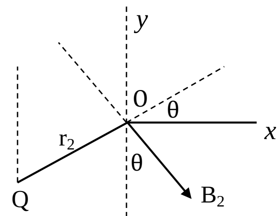
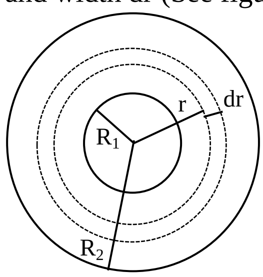


Figure (2)

Ans: b

31. Let σ be charge per unit area of the annular disc. Take a ring of radius r and width dr (See figure)
Area of ring $dA = 2\pi r\ dr$;
charge on the ring is dq $= \sigma dA = \sigma 2\pi r\, dr$
Current due to rotation $dI = dq.f = \sigma 2\pi r\, dr.f$
∴ Magnetic moment of rotating charge $= \pi r^2 dI$
Total magnetic moment of the annular disc is $M = \int \pi r^2 dI$



$$M = \int \pi r^2 dq\, f = \int_{R_1}^{R_2} \pi r^2 \sigma .2\pi r\, dr. f$$

$$= \sigma .2\pi^2 f \int_{R_1}^{R_2} r^3 dr = \sigma .2\pi^2 f \, \frac{R_2^4 - R_1^4}{4} = \frac{\sigma .\pi^2 f}{2}\left(R_2^2 + R_1^2\right)\left(R_2^2 - R_1^2\right)$$

Now $q = \sigma(\pi R_2^2 - \pi R_1^2)$ or $\sigma = \dfrac{q}{\pi(R_2^2 - R_1^2)}$ giving $M = \dfrac{\pi q f\,(R_1^2 + R_2^2)}{2}$

Ans: a

32. When R and C are in series, the impedance is $Z_{series} = R - j\dfrac{1}{\omega C}$ and

when R and C are in parallel the impedance $Z_{||}$ is obtained by $\dfrac{1}{Z_{||}} = \dfrac{1}{R} + \dfrac{1}{-j\dfrac{1}{\omega C}} = \dfrac{1}{R} + j\omega C$

Thereby $\left|Z_S\right| = \sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}$ and $\left|\dfrac{1}{Z_{||}}\right| = \sqrt{\dfrac{1}{R^2} + \omega^2 C^2}$ or

$Z_S = \dfrac{\sqrt{1+\omega^2 C^2 R^2}}{\omega C}$ and $\left|Z_{||}\right| = \dfrac{R}{\sqrt{1+\omega^2 C^2 R^2}}$

Given that $Z_S = 2Z_{||} \Rightarrow \dfrac{\sqrt{1+\omega^2 C^2 R^2}}{\omega C} = \dfrac{2R}{\sqrt{1+\omega^2 C^2 R^2}}$

Or $1+\omega^2 C^2 R^2 = 2R\omega C \Rightarrow \omega^2 C^2 R^2 - 2\omega CR + 1 = 0$

Or $(\omega CR - 1)^2 = 0 \Rightarrow \omega = \dfrac{1}{CR} \Rightarrow f = \dfrac{1}{2\pi CR}$

Ans: b

33. According to the Einstein's equation for photoelectric effect $h\upsilon = \phi + \dfrac{1}{2}mv^2$

If stopping potential is V and $\upsilon = \dfrac{c}{\lambda}$: we have $\dfrac{hc}{\lambda} = \phi + eV$. For $\lambda = \lambda_0$, $\dfrac{1}{2}mv^2 = 0$

$\therefore \dfrac{hc}{\lambda} = \dfrac{hc}{\lambda_0} + eV$ …… (1)

For $\lambda_1 = 300\,nm$ $V_1 = 1.8\,V$ therefore $\dfrac{hc}{300\,nm} = \dfrac{hc}{\lambda_0} + e(1.8)$ ….. (2)

For $\lambda_2 = 400\,nm$ $V_2 = 0.9\,V$ therefore $\dfrac{hc}{400\,nm} = \dfrac{hc}{\lambda_0} + e(0.9)$ ….. (3)

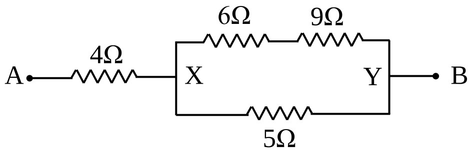
Multiply equation (3) by 2 and subtracting from equation (2) we get

$$hc\left(\frac{1}{200} - \frac{1}{300}\right) = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = 600\,nm$$

Ans: c

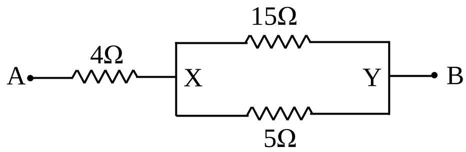
34. If the current in 5 Ω is I then the current in 15 (= 6 + 9) Ω is $\frac{I}{3}$ and that in 4 Ω is $I + \frac{I}{3} = \frac{4I}{3}$

Power dissipated in $5\Omega = I^2 \times 5 = 7.2\ W$ given



$I^2 = \frac{7.2}{5} = 1.44 \Rightarrow I = 1.2\,A$

$\therefore$ The current in $6\Omega = \frac{I}{3} = 0.4\,A$



Power in $6\Omega = 0.4^2 \times 6 = 0.96\ W$: a is wrong

Potential difference between A and X is $\frac{4I}{3} \times 4 = \frac{16I}{3}$

Potential difference between X and Y is $I \times 5 = 5I$

Potential difference between A and B is $\frac{16I}{3} + 5I = \frac{31I}{3}$

$= \frac{31}{3} \times 1.2 = 31 \times 0.4 = 12.4V$

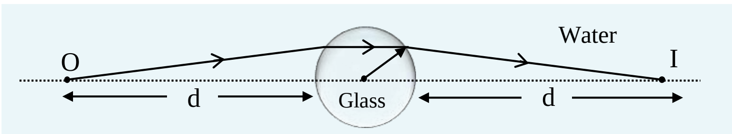
Ans: b

35. By symmetry and reversibility of the path of the light, the ray in the glass sphere should be parallel to the principal axis

For the refraction at the first surface, $\frac{{}_a\mu_g}{v} - \frac{{}_a\mu_w}{u} = \frac{{}_a\mu_g - {}_a\mu_w}{+r}$

${}_a\mu_g = \frac{3}{2}, \quad {}_a\mu_w = \frac{4}{3}, \quad u = -d,\ v = \infty$ gives

$\frac{\frac{4}{3}}{d} = \frac{\frac{3}{2} - \frac{4}{3}}{+r} = \frac{1}{6r}$



$d = \frac{4}{3} \times 6r = 8r$

Ans: d

36. The capacity of the parallel plate capacitor is $C = \frac{k\in_0 A}{d} = \frac{k\in_0 AE}{Ed} = \frac{k\in_0 AE}{V}$

Or $A = \frac{\pi D^2}{4} = \frac{CV}{k\in_0 E} \Rightarrow D^2 = \frac{4CV}{\pi k\in_0 E} = \frac{4\times 72\times 10^{-9}\times 4\times 10^3}{3.14\times 2.5\times 8.85\times 10^{-12}\times 18\times 10^6}$

$\Rightarrow D = \sqrt{\frac{4\times 72\times 4}{3.14\times 2.5\times 8.85\times 18}} = \sqrt{0.921} = 0.96\ m = 96\ cm$

Ans: d

37. The moment of inertia of a solid sphere about a tangent is $I_T = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$.

Hence by the principle of conservation of angular momentum, one can write

$\frac{2}{5}mR^2\left(\frac{v}{R}\right) + mv\left(R - 0.2R\right) = \frac{7}{5}mR^2\omega \Rightarrow \omega = \frac{6v}{7R}$

Ans: c

38. Let us consider a square loop of side ℓ carrying current i. The magnetic field produced by one side at point P, at a distance x on the axis of the loop is $B = \frac{\mu_0 i}{4\pi\sqrt{x^2 + \frac{\ell^2}{4}}}\left[2\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}+\frac{\ell^2}{4}}}\right]$ directed perpendicular to the side AB as well as MP. M is the midpoint of side AB.

The component of this B along the axis is $B\cos(90-\theta)$

$$B\sin\theta = \frac{\mu_0 i}{4\pi\sqrt{x^2 + \frac{\ell^2}{4}}}\left[2\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}+\frac{\ell^2}{4}}}\right]\times\left[\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}}}\right]$$

Due to the four sides, this value will become 4 times. So the resultant magnetic field is

$$B = \frac{\mu_0 i}{4\pi}\times 4\frac{1}{\sqrt{x^2 + \frac{\ell^2}{4}}}\left[2\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}+\frac{\ell^2}{4}}}\right]\times\left[\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}}}\right] \quad \text{or} \quad B = \frac{\mu_0 i}{4\pi}\frac{4\times 2\sqrt{2}\,\ell^2}{\left(4x^2+\ell^2\right)\sqrt{2x^2+\ell^2}}$$

Ans: c

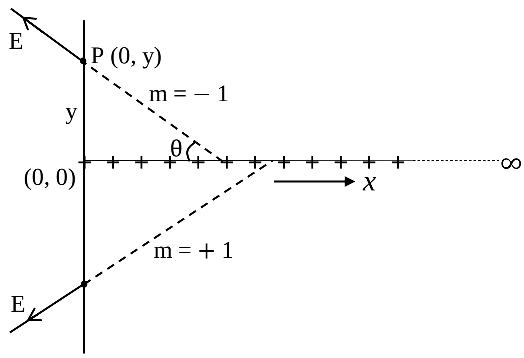
39. The electric field dE at point P (0, y) due to an element of charge $\lambda\, dx$ of length dx at a point (x, 0) is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{\left(\sqrt{x^2+y^2}\right)^2}$$

This dE is directed along the line joining the point (x, 0) and the point (0, y). This dE can be resolved in components along x – axis and along y – axis, the components being

$$dE\cos\theta = \frac{1}{4\pi\varepsilon}\frac{\lambda dx}{x^2+y^2}\frac{x}{\sqrt{x^2+y^2}} = \frac{\lambda}{4\pi\varepsilon_0}\frac{x\,dx}{\left(x^2+y^2\right)^{\frac{3}{2}}} \quad \text{and}$$

$$dE\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2+y^2}\frac{y}{\sqrt{x^2+y^2}} = \frac{\lambda}{4\pi\varepsilon_0}\frac{y\,dx}{\left(x^2+y^2\right)^{\frac{3}{2}}}$$



Now integrating over the limits $x = 0$ to $x = \infty$

we get $E_x = \frac{\lambda}{4\pi\varepsilon_0 y}\int_{\frac{\pi}{2}}^{0}\cos\theta\, d\theta = -\frac{\lambda}{4\pi\varepsilon_0 y}$

directed parallel to the line of charge along – x and

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 y}\int_{\frac{\pi}{2}}^{0}\sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 y}$$

directed perpendicular to the line of charge along + y if y is positive and along – y if y is negative.

The resultant field is $E = \sqrt{E_x^2 + E_y^2} = \frac{\lambda}{4\pi\varepsilon_0 y}\sqrt{1+1} = \frac{\lambda}{4\pi\varepsilon_0 y}\sqrt{2}$

The resultant E is always directed along a line subtending angle 45° with the line of charge or the negative direction of x axis, the gradient being $m = -1$ when y is positive and $m = +1$ if y is negative

Ans: d

40. The electric field at a point P near an infinite sheet of charge of surface charge density σ is

$E = \frac{\sigma}{2\in_0}$ directed away and perpendicular to the plane of the sheet.

To express the result on the two sides of a uniformly charged sheet lying in the xy-plane, one can write it as $E = \frac{z}{|z|}\frac{\sigma}{2\in_0}\hat{k}$. Similarly for a sheet in yz-plane $E = \frac{x}{|x|}\frac{-\sigma}{2\in_0}\hat{i}$.

The two fields being mutually perpendicular, the resultant will be

$$E = -\frac{x}{|x|}\frac{\sigma}{2\in_0}\hat{i} + \frac{z}{|z|}\frac{\sigma}{2\in_0}\hat{k} \quad \text{or} \quad E = \frac{\sigma}{2\in_0}\left(-\frac{x}{|x|}\hat{i} + \frac{z}{|z|}\hat{k}\right)$$

Ans: c

41. The electric field of a spherical distribution of charge is

$$E = \frac{1}{4\pi\in_0}\frac{\int_0^r \rho_0\left(1-\frac{\xi}{R}\right)\times 4\pi\xi^2 d\xi}{r^2} \quad \text{for} \quad r \le R \text{ and}$$

$$E = \frac{1}{4\pi\in_0}\frac{\int_0^R \rho_0\left(1-\frac{\xi}{R}\right)\times 4\pi\xi^2 d\xi}{r^2} \quad \text{for} \quad r \ge R$$

For $r \le R$; $E = \frac{\rho_0}{\in_0}\frac{\int_0^r\left(\xi^2 d\xi - \frac{\xi^3 d\xi}{R}\right)}{r^2} = \frac{\rho_0}{\in_0}\frac{\left(\frac{\xi^3}{3}-\frac{\xi^4}{4R}\right)_0^r}{r^2} = \frac{\rho_0}{\in_0}\frac{\left(\frac{r^3}{3}-\frac{r^4}{4R}\right)}{r^2} = \frac{\rho_0 r}{\in_0}\left(\frac{1}{3}-\frac{r}{4R}\right)$

Thus both options (a) and (b) are wrong.

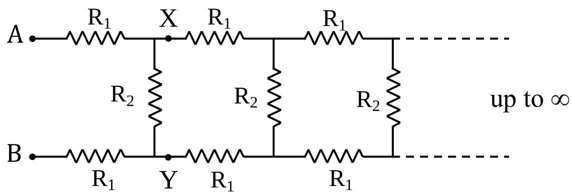
E(r) is maximum when $\frac{dE}{dr} = 0 \Rightarrow \frac{\rho_0}{\in_0}\left[\frac{1}{3}-\frac{r}{2R}\right] = 0 \Rightarrow r = \frac{2}{3}R$; option c is correct

Thus $E_{max} = \frac{\rho_0}{\in_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)_{r=\frac{2}{3}R} = \frac{\rho_0}{\in_0}\left(\frac{2R}{9}-\frac{4R^2}{4\times 9R}\right) = \frac{\rho_0 R}{9\in_0}$ and not $\frac{\rho_0 R}{3\in_0}$; d is wrong

Ans: c

42. Let us consider that the equivalent resistance is $R_{eff} = R$ across A B.

It is the same R as across XY due to infinite repetitions



Or $2R_1 + \frac{RR_2}{R+R_2} = R$

Or $2RR_1 + 2R_1R_2 + RR_2 = R^2 + RR_2$

Or $R^2 - 2RR_1 - 2R_1R_2 = 0$

$$R = \frac{2R_1 \pm \sqrt{4R_1^2 + 8R_1R_2}}{2}$$

Take + sign (− ve sign gives a negative value of R which is not acceptable)

$$R = R_1 + \sqrt{R_1^2 + 2R_1R_2}$$

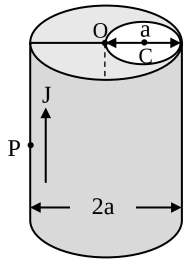
Ans: a

43. Axis of the cylinder is through point O. We have to calculate the magnetic field at point P on the surface of the cylinder. It may be assumed that inside the cavity there is a superposition of two current densities + J and – J. i.e. there are two opposite currents.

(1) A current $I_1 = J\pi a^2$ in the whole of the cylinder which produces a magnetic field at P as

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{a} = \frac{\mu_0}{4\pi}\frac{2J\pi a^2}{a} = \frac{\mu_0 Ja}{2}$$ and

(2) A current $I_2 = J\left(\pi\frac{a^2}{4}\right)$ in the cavity in opposite direction which produces a magnetic field at P



as $B_2 = \frac{\mu_0}{4\pi}\frac{2I_2}{\frac{3}{2}a} = \frac{\mu_0}{4\pi}\frac{2J\pi\frac{a^2}{4}}{\frac{3}{2}a} = \frac{\mu_0 Ja}{12}$

$\therefore$ Resultant field at P is $B = B_1 - B_2 = \frac{5}{12}\mu_0 Ja$

Ans: d

44. The magnetic field exerts a force $\vec{F} = i\vec{\ell}\times\vec{B}$. This force tends to turn the rod clockwise and may be considered to be acting at the center of the rod perpendicular to its length. Hence the clockwise torque

is $\tau = i\ell B \times \frac{\ell}{2} = 10\times\frac{0.2^2}{2}\times 0.5 = 0.1\,Nm$ Statement (1) is correct

The spring exerts horizontal force Kx which exerts an anti-clockwise torque $= Kx\times\ell\sin 53°$

In equilibrium $= Kx\times\ell\sin 53° = i\ell\times B\times\frac{\ell}{2}$ therefore $x = \frac{iB\frac{\ell}{2}}{K\sin 53°}$

Therefore the energy stored in the spring is $\frac{1}{2}Kx^2 = \frac{i^2\ell^2\times B^2}{2\times 4K\times\left(\frac{4}{5}\right)^2} = \frac{25\times10^2\times0.2^2\times0.5^2}{128\times5} = 0.039\,J$

Statement (2) is correct

Ans: d

45. The displacement current density through a dielectric is $J_d = K\in_0\frac{dE}{dt}$ therefore

$i_d = K\in_0\frac{d}{dt}\int E.dS = K\in_0\frac{d\phi}{dt}$; $i_d = K\in_0\frac{d\phi}{dt}$

or $12.5\times10^{-12} = K\times 8.85\times10^{-12}\times 8000\times 4t^3$ substituting t = 20 × 10^{-3} s we get $K = 5.517 \cong 5.52$

Ans: b

46. Considering f_1 and f_2 as the two focal lengths of a lens in object space and image space respectively.

The focal length of a lens is given by $\frac{\mu_2}{f_2} = \frac{\mu-\mu_1}{R_1} - \frac{\mu-\mu_2}{R_2}$

For refraction at the first curved surface $\frac{\mu_1}{v_1} - \frac{4/3}{\infty} = \frac{\mu_1 - 4/3}{+20}$ ….. (1)

For refraction at the second curved surface $\frac{\mu_2}{v_2} - \frac{\mu_1}{v_1} = \frac{\mu_2-\mu_1}{-20}$ ….. (2)

For refraction at the third curved surface $\frac{4/3}{v_3} - \frac{\mu_2}{v_2} = \frac{4/3 - \mu_2}{+20}$ ….. (3)

Adding the three $\frac{4/3}{v_3} = \frac{\mu_1 - 4/3}{+20} + \frac{\mu_2 - \mu_1}{-20} + \frac{4/3 - \mu_2}{+20}$ or

$$\frac{4/3}{f} = \frac{1}{20}\left[\mu_1 - 4/3 - (\mu_2 - \mu_1) + 4/3 - \mu_2\right] \Rightarrow \frac{4}{3f} = \frac{2}{20}\left[\mu_1 - \mu_2\right] \text{ or}$$

$$\Rightarrow \frac{4}{3 \times 24} = \frac{1}{10}\left[\mu_1 - \mu_2\right] \Rightarrow (\mu_1 - \mu_2) = \frac{5}{9}$$

Ans: d

Alternatively:-

Let μ_1' and μ_2' be the refractive indices of flint and crown glass with respect to water, respectively, such that $\mu_1' = \frac{\mu_1}{\frac{4}{3}}$ and $\mu_2' = \frac{\mu_2}{\frac{4}{3}}$. Also the focal length of the two lenses

$\frac{1}{f_1} = (\mu_1' - 1) \times \frac{2}{R}$ and $\frac{1}{f_2} = (\mu_2' - 1) \times \frac{2}{R}$. All the radii are equal and R = 20 cm (given)

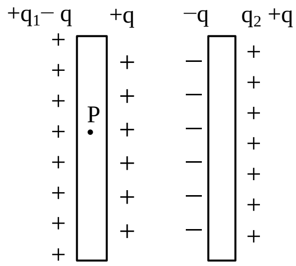
Thereby the combined focal length f can be obtained as

$$\frac{1}{f} = \frac{1}{f_1} + \left(-\frac{1}{f_2}\right) = \frac{1}{f_1} - \frac{1}{f_2} = (\mu_1' - 1) \times \frac{2}{R} - (\mu_2' - 1) \times \frac{2}{R} \text{ or } \frac{1}{f} = (\mu_1' - \mu_2') \times \frac{2}{R}$$

$$\frac{1}{24} = \left(\frac{\mu_1}{4/3} - \frac{\mu_2}{4/3}\right) \times \frac{2}{20} \Rightarrow \mu_1 - \mu_2 = \frac{5}{9}$$

Ans: d

47. Considering charge $+q$ and $-q$ on parallel interior faces, the charge on the exterior faces of the two plates will be $+q_1 - q$ on extreme left & $+q_2 + q$ on extreme right.



The electric field E at any point P within the first metallic plate is

$$E = +\frac{q_1 - q}{2\varepsilon_0 A} - \frac{q}{2\varepsilon_0 A} + \frac{q}{2\varepsilon_0 A} - \frac{+q_2 + q}{2\varepsilon_0 A} = 0$$

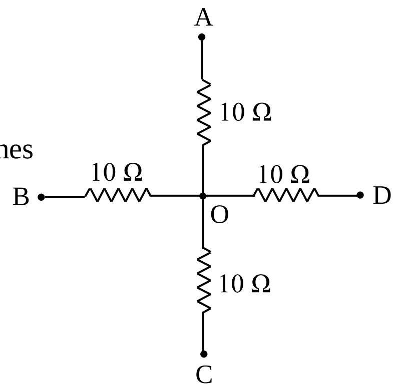
The electric field inside a metal is zero. This gives

$$+q_1 - q - q + q - q_2 - q = 0 \Rightarrow q_1 - q_2 = 2q \quad \text{or} \quad q = \frac{q_1 - q_2}{2}$$

writing now $q = CV \Rightarrow \frac{q_1 - q_2}{2C} = V$

Ans : b

48. If the potential at the node O is V_1 then by Kirchoff's law, the current coming from D to O shall be equally divided in the other three branches namely OA, OB and OC.



Hence $\frac{2 - V_1}{10} = \frac{V_1 - 1}{10} + \frac{V_1 - 1}{10} + \frac{V_1 - 1}{10} \Rightarrow V_1 = \frac{5}{4} volt$

Ans: c

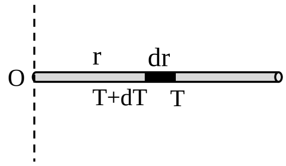
A – 2

ANY NUMBER OF OPTIONS (4, 3, 2 or 1) MAY BE CORRECT

MARKS WILL BE AWARDED ONLY IF ALL THE CORRECT OPTIONS ARE BUBBLED AND NO INCORRECT.

49. Let there be a small element of length dr at a distance r from the axis of rotation on the rod rotating about a vertical axis through the end at O. Let T and T + dT be tension at the two ends of dr as shown.

The mass of the element of small length dr is $dm = \rho A dr$



Centripetal force acting on this element $= dm.\omega^2 r = (\rho A dr).\omega^2 r$

This force is provided by tension $T - (T + dT) = \rho A \omega^2 r dr$

$\frac{dT}{dr} = -\rho A \omega^2 r$.

Integrating partially $T(r) = -\rho A \omega^2 \frac{r^2}{2} + C$ ….. (1)

At the extreme end, $r = \ell = 1m$ and T = 0 thereby $C = \rho A \omega^2 \frac{\ell^2}{2}$.

Substituting in equation (1) $T(r) = \frac{\rho A \omega^2}{2}(\ell^2 - r^2)$

Taking $\rho = 10^4 kg/m^3$, $A = 2\times10^{-6} m^2$ and $\omega = 400 \ rad/s$ we get

$T(r) = 1600(\ell^2 - r^2) = 1600(1^2 - r^2) = 1600(1 - r^2)$: b is wrong

Further at $r = \frac{\ell}{2} = \frac{1}{2} = 0.5 \ m$

$T = 1600(1 - 0.5^2) = 1200 N$: a is correct

Stress at $r = 0.5 \ m$ (midpoint) is $\frac{T}{A} = \frac{1200}{2\times10^{-6}} = 6\times10^8 \frac{N}{m^2}$: c is wrong

$Y = \frac{stress}{strain} = \frac{\frac{T}{A}}{\frac{d\xi}{dr}} = \frac{T\,dr}{A\,d\xi}$ where $d\xi$ is elongation of length dr

Therefore $d\xi = \frac{T}{AY}dr = \frac{1600(1-r^2)\,dr}{2\times10^{-6}\times2\times10^{11}} = 4\times10^{-3}(1-r^2)\,dr$

The total elongation $\xi = \int_0^{\xi} d\xi = 4\times10^{-3}\int_0^1 (1-r^2)\,dr = 4\times10^{-3}\left[r - \frac{r^3}{3}\right]_0^1 = 4\times10^{-3}\left(1 - \frac{1}{3}\right)$

$$\xi = \frac{8}{3}\times10^{-3} \ m = \frac{8}{3} \ mm$$; d is correct

Ans: a, d

50. Velocity of transverse wave on a stretched string (rope) is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{8}{\frac{0.02}{1}}} = 20\,ms^{-1}$

Frequency $f = 100\,Hz$

$\therefore \lambda$ of travelling wave $= \frac{v}{f} = \frac{20}{100} \ m = 20 \ cm$: a is correct

Maximum acceleration $= a\omega^2$ where the amplitude $a = 0.02 \ m$ and $\omega = 2\pi f = 200\pi$

Therefore $acc_{max} = 0.02\times(200\pi)^2 = 800\pi^2 \ ms^{-2}$ not 800 : b is wrong

Let the equation of the wave be $y = a\ \sin(kx - \omega t + \phi)$

At $x = 0$ at the time t = 0, we get $y = a\ \sin\phi$ means $-a = a\ \sin\phi \Rightarrow \phi = \frac{3\pi}{2}$

given that $a = 0.02\,m$

substituting theses values we get

$$y = a\ \sin\left(kx - \omega t + \frac{3\pi}{2}\right) = -a\ \cos(kx - \omega t)$$

Also $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.2} = 10\,\pi$ and $\omega = 2\pi f = 2\pi \times 100 = 200\,\pi$

Therefore $y = -0.02\cos(10\pi x - 200\pi t)$: c is wrong

When tension is not changed, the wave velocity v will also not change. However changing the frequency to f = 200 we get

$\lambda = \frac{20}{200} = 0.1m = 10\,cm$, which is half of the earlier value of wavelength : d is correct

Ans: a, d

51. Let the number of nuclei at time t be N

The rate of decay of N is $\frac{dN}{dt} = -\lambda N$ and the rate at which the nuclei are being produced is α (given)

$$\therefore \frac{dN}{dt} = \alpha - \lambda N \Rightarrow \frac{dN}{\alpha - \lambda N} = dt \quad \ldots\ldots (1)$$

On integration $\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + C$

At t = 0, N = N_0 therefore $\frac{\log_e(\alpha - \lambda N_0)}{-\lambda} = C$ substituting

Or $\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + \frac{\log_e(\alpha - N_0)}{-\lambda}$

$$\log_e \frac{\alpha - \lambda N}{\alpha - \lambda N_0} = -\lambda t \quad \Rightarrow \frac{\alpha - \lambda N}{\alpha - \lambda N_0} = e^{-\lambda t}$$

$$\alpha - \lambda N = (\alpha - \lambda N_0)e^{-\lambda t} \Rightarrow \lambda N = \alpha - (\alpha - \lambda N_0)e^{-\lambda t}$$

$N(t) = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right]$: a is wrong

If $\alpha = \lambda N_0$ we get $N(t) = \frac{\alpha}{\lambda} = \frac{\lambda N_0}{\lambda} = N_0$ = constant for all t ; b is correct

As $\underset{t\to\infty}{Lt}\ e^{-\lambda t} = 0$ and $\alpha = 2\lambda N_0$ then we get $N(t) = 2N_0$; c is correct

Again if $\alpha = 2\lambda N_0$ we have

$$N(t) = \frac{1}{\lambda}\left\{\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right\} = \frac{1}{\lambda}\left\{2\lambda N_0 - \lambda N_0\, e^{-\lambda t}\right\}$$

Or $N(t) = 2N_0 - N_0 e^{-\lambda t} = N_0\left\{2 - e^{-\lambda t}\right\}$ further using $\lambda = \frac{0.693}{T} = \frac{\ln 2}{T}$

At t = T where T is half-life we have $e^{-\lambda t} = e^{-\lambda T} = e^{-\frac{\ell n 2}{T}.T} = e^{-\ell n 2} = e^{\ell n\left(\frac{1}{2}\right)} = \frac{1}{2}$

$\therefore$ after one half-life $N(t) = N_0\left\{2 - \frac{1}{2}\right\} = \frac{3}{2}N_0$: d is correct

Ans: b, c, d

52. In Young's double slit interference experiment, the resultant intensity

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

At central maximum the path difference hence the phase difference $\phi = 0, \cos\phi = 1$ this gives maximum intensity

So $I_{\max} = I + I + 2I \times \cos 0 \Rightarrow I_{\max} = 4I$ ….. (1)

When the thin transparent film has been introduced in front of the second slit

$I_{\text{Resultant}} = I + I + 2I\cos\phi = \frac{I_{max}}{2} = 2I$ (given) $\Rightarrow \cos\phi = 0$ or $\phi = \frac{\pi}{2}$

It means a phase difference of $\frac{\pi}{2}$ has been introduced which corresponds to a path difference of

$\Delta = \frac{\lambda}{2\pi} \times$ phases difference or $\Delta = \frac{\lambda}{2\pi} \times \frac{\pi}{2} = \frac{\lambda}{4}$

Hence $\mu t - t = \frac{\lambda}{4}$ or $t(\mu - 1) = \frac{\lambda}{4} \Rightarrow \mu - 1 = \frac{\lambda}{4t} = \frac{600\times10^{-9}}{4\times250\times10^{-9}} = 0.6 \Rightarrow \mu = 1.6$: a is correct

Also the film thickness $t = \frac{p\lambda}{\mu - 1} = \frac{x}{\overline{X}}\frac{\lambda}{\mu - 1}$ where p is the number of shifted fringes

$\Rightarrow \overline{X} = \frac{x}{t}\frac{\lambda}{(\mu-1)} = \frac{0.5\times10^{-3}\times600\times10^{-9}}{250\times10^{-9}\times0.6} = 2\times10^{-3} = 2\,mm$; b is correct

The film placed in front of S_2 increases the optical path S_2O. To compensate the increase of path, point O must go down hence O' should be below O and not above O : c is wrong

Further the fringe width is $\overline{X} = \frac{D\lambda}{d}$ and the angular fringe width $\beta = \frac{\overline{X}}{D} = \frac{\lambda}{d} = 0.1^\circ$ (given)

$\Rightarrow \frac{\overline{X}}{D} = 0.1^\circ = \frac{1}{10}\times\frac{\pi}{180}$ radian or $D = \frac{1800\overline{X}}{\pi} = \frac{1800\times2\times10^{-3}}{\pi} = 1.146 \cong 1.15\,m$; d is correct

Ans: a, b, d

53. The electric field inside a uniformly charged non conducting sphere of radius 'a' is

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{4Q}{a^3}\vec{r}$; a is correct

The energy stored is $U = \int_0^a \frac{1}{2}\epsilon_0 E^2 4\pi r^2 dr = \frac{1}{2}\epsilon_0 4\pi \int_0^a \left(\frac{1}{4\pi\epsilon_0}\frac{4Q}{a^3} r\right)^2 r^2 dr = \frac{2}{\pi\epsilon_0}\frac{Q^2}{a^6}\int_0^a r^4\,dr$

$= \frac{2Q^2}{\pi\epsilon_0}\frac{1}{a^6}\left(\frac{r^5}{5}\right)\Big|_0^a = \frac{2}{5\pi\epsilon_0}\frac{Q^2}{a^6}(a^5 - 0) = \frac{2}{5\pi\epsilon_0}\frac{Q^2}{a}$; d is correct

The charge +4Q induces charge − 4Q on the inner surface of the shell and thereby +4Q on the outer surface of the shell. : b is wrong

Thus the net charge on the outermost surface is +4Q – 2Q = +2Q ; c is correct

Ans: a, c, d

54. Energy of the electron in the n[th] Bohr orbit of a hydrogen like atom

$E_n = -\frac{13.6Z^2}{n^2}\,eV \Rightarrow E_2 = -\frac{13.6Z^2}{2^2}$ and $E_3 = -\frac{13.6Z^2}{3^2}$

$E_3 - E_2 = 13.6Z^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 13.6Z^2\left[\frac{9-4}{36}\right] = \frac{5\times13.6\times Z^2}{36} = 47.25\,eV \Rightarrow Z = 5$: a is correct

$E_4 - E_3 = 13.6Z^2\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = 13.6Z^2\left(\frac{16-9}{144}\right) = \frac{7}{144}\times13.6\times25 = 16.528 = 16.53\,eV$: b is correct

To remove the electron completely from first Bohr orbit (means the ionization energy) is

$$\frac{13.6\ Z^2 e}{1^2} = h\upsilon = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{13.6 Z^2 e}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{13.6\times25\times1.6\times10^{-19}} = \frac{6.63\times3}{13.6\times25\times1.6}\times10^{-7} = 0.0365\times10^{-7} = 36.56\times10^{-10} = 36.56\,\text{Å}$$: c is correct

The angular momentum of an electron in n[th] orbit is

$$J = mvr = \frac{nh}{2\pi} = \frac{2\times6.63\times10^{-34}}{2\pi}Js = 2.11\times10^{-34}Js$$: d is wrong

Ans: a, b, c

55. As shown in the figure, during the process AB, $\frac{\rho}{P}$ is constant so to say $\frac{1}{PV}$ is constant or PV is constant means the process is Isothermal hence the Work done by the gas is

$W_{AB} = nRT\ \ell n\frac{V_2}{V_1} = RT\ell n\frac{\rho_1}{\rho_2} = -\frac{MP_0}{\rho_0}\ell n\,2$ The negative sign shows that the work is done **on** the gas and **not by** the gas. Hence the work done on the gas is $= \frac{MP_0}{\rho_0}\ell n2$; a is correct

As a result the gas compresses and rejects heat $= \frac{MP_0}{\rho_0}\ell n2$

The work done in the isobaric process BC is $W_{BC} = 2P_0\left(\frac{M}{\rho_0} - \frac{M}{2\rho_0}\right) = \frac{MP_0}{\rho_0}$; b is wrong

No work is done in the isochoric process CA

The heat rejected by the gas is $dQ = dQ_{AB} + dQ_{BC} + dQ_{CA}$

In the Isothermal process AB the heat taken by the gas is equal to the work done on the gas hence

$$dQ_{AB} = W_{AB} + dU_{AB} = -\frac{MP_0}{\rho_0}\ell n\,2 + 0 = -\frac{MP_0}{\rho_0}\ell n\,2$$

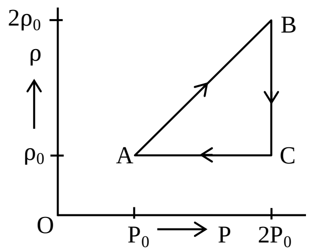
In the Isobaric process BC ($C_P = \frac{5R}{2}$ for monoatomic gas) where the gas is expanding the heat absorbed is $dQ_{BC} = C_P dT = \frac{5}{2}R[T_C - T_B] = \frac{5}{2}R[2T_A - T_A] = \frac{5}{2}RT_A = \frac{5MP_0}{2\rho_0}$

The process CA is isochoric therefore no work is done i.e. $W_{CA} = 0$ while the heat added is

$dQ_{CA} = C_V dT = \frac{3}{2}R[T_A - T_C] = \frac{3}{2}R[T_A - 2T_A] = -\frac{3}{2}RT_A = -\frac{3MP_0}{2\rho_0}$ means heat is taken away from the gas i.e. the heat is rejected by the gas

Thereby the Net heat rejected by the gas in the complete cycle ABCA is

$$dQ = \frac{MP_0}{\rho_0}\ell n\,2 + \frac{3MP_0}{2\rho_0} = \frac{MP_0}{\rho_0}\left(\frac{3}{2} + \ell n\,2\right)$$: d is wrong



The work done $W = W_{AB} + W_{BC} + W_{CA}$

$$= -\frac{MP_0}{\rho_0}\ell n2 + \frac{MP_0}{\rho_0} + 0 = \frac{MP_0}{\rho_0}\left(1 - \ell n\,2\right)$$

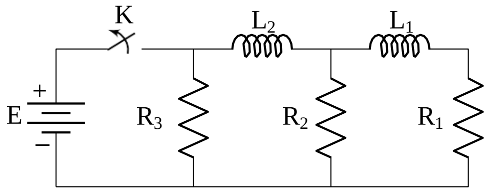
Heat is absorbed by the gas only the processes BC and is $dQ_{BC} = \frac{5MP_0}{2\rho_0}$

Efficiency of the cycle is $\eta = \dfrac{\text{Work done}}{\text{Heat absorbed}} = \dfrac{\frac{MP_0}{\rho_0}(1-\ell n\,2)}{\frac{5MP_0}{2\rho_0}} = \dfrac{2}{5}(1-\ell n\,2)$ c is correct

Ans: a, c

56. Closing the key for a long time, the current in LR circuit reaches a maximum (steady current) and inductor becomes ineffective. In the given circuit the steady current through each resistor is $I = \dfrac{E}{R}$.

When the key is suddenly opened the current decays starting from its steady state value I hence the current through R_1 shall be instantaneously I downward through R_2 also it would be I downward (the inductance will oppose the decay of current in R_1 and R_2) while through R_3 the two currents join and it will be 2I upwards



Ans: a, b, c

57. The potential energy function $U(x) = \dfrac{\alpha}{x^2} - \dfrac{\beta}{x}$ and $\dfrac{\alpha}{\beta} = x_0$ (given).

Now substituting $\beta = \dfrac{\alpha}{x_0}$; $U(x) = \dfrac{\alpha}{x^2} - \dfrac{\alpha}{x\,x_0} = \dfrac{\alpha}{x_0^2}\left[\left(\dfrac{x_0}{x}\right)^2 - \dfrac{x_0}{x}\right]$; a is correct.

Further we know that the force $F(x) = -\dfrac{dU(x)}{dx} \Rightarrow m\dfrac{dv}{dt} = -\dfrac{dU(x)}{dx}$ or $m\dfrac{dv}{dx}\dfrac{dx}{dt} = -\dfrac{dU(x)}{dx}$

$\Rightarrow mv\,dv = -\dfrac{dU(x)}{dx}dx$ or $mv\,dv = -dU(x)$ integrating $m\dfrac{v^2}{2} = -U(x) = \dfrac{\alpha}{x_0^2}\left[\dfrac{x_0}{x} - \left(\dfrac{x_0}{x}\right)^2\right]$

Since the particle starts from rest at $x = x_0$, the constant of integration is zero.

$v^2 = \dfrac{2\alpha}{mx_0^2}\left[\dfrac{x_0}{x} - \left(\dfrac{x_0}{x}\right)^2\right]$ or $v = \sqrt{\dfrac{2\alpha}{mx_0^2}}\left[\dfrac{x_0}{x} - \left(\dfrac{x_0}{x}\right)^2\right]^{\frac{1}{2}}$; b is correct

The velocity will be maximum when $\dfrac{dv}{dx} = 0$ means $\dfrac{dU(x)}{dx} = 0$ i.e. when $-\dfrac{2\alpha}{x^3} + \dfrac{\beta}{x^2} = 0$ or

$\dfrac{2\alpha}{x} - \beta = 0$ or $x = \dfrac{2\alpha}{\beta} = 2x_0$

therefore $v_{\max} = \sqrt{\dfrac{2\alpha}{mx_0^2}\left[\dfrac{x_0}{2x_0} - \left(\dfrac{x_0}{2x_0}\right)^2\right]} = \sqrt{\dfrac{2\alpha}{mx_0^2}\left[\dfrac{1}{2} - \left(\dfrac{1}{4}\right)\right]} = \sqrt{\dfrac{\alpha}{2mx_0^2}}$; c is correct

Further the total energy of the particle is $E = KE + PE = \dfrac{1}{2}mv^2 + U(x) = -U(x) + U(x) = 0$

hence the total energy of the particle is zero ; d is correct

Ans: a, b, c, d

58. When the block C collides elastically with block A of equal mass M, by the exchange of velocity, the block C will stop and block A will move with velocity v. In-turn the block A will compress the spring, therefore block B will start moving. As long as the velocity of block A is more than that of B, the spring will keep on getting compressed. At the moment the spring is compressed to the maximum, both the blocks A and B will have equal velocities say v'

Initial kinetic energy of mass M is $\text{KE} = \frac{1}{2} M v^2$

Initial momentum = $M v$

At the maximum compression of the spring, the momentum $= M v' + 2M v' = 3M v'$

By the conservation of momentum $M v = 3M v' \Rightarrow v' = \frac{v}{3}$

Therefore the kinetic energy of blocks A and B together, after the collision is

$KE = \frac{1}{2}(M + 2M) v'^2 = \frac{1}{2} 3M \left(\frac{v}{3}\right)^2 = \frac{M v^2}{6}$: b is correct

Thereby the loss of KE $= \frac{1}{2} M v^2 - \frac{M v^2}{6} = \frac{M v^2}{3}$

This loss will appear as potential energy of the spring hence $PE = \frac{1}{2} k x_{\max}^2$

Thereby $\frac{1}{2} k x_{\max}^2 = \frac{M v^2}{3}$

$x_{\max}^2 = \frac{2}{3} \frac{M}{k} v^2$

$x_{\max} = v\sqrt{\frac{2M}{3k}}$: a is wrong

At the maximum compression of the spring, the blocks A and B have same velocity v' obtained by conservation of momentum: c is wrong

Finally the blocks A, B and the spring will oscillate as a two-body oscillation with time period $T = 2\pi\sqrt{\frac{\mu}{k}}$ where the reduced mass is $\mu = \frac{M \times 2M}{M + 2M} = \frac{2}{3} M$

Therefore $T = 2\pi\sqrt{\frac{2M}{3k}}$

Time required for reaching the maximum compression is $= \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{2M}{3k}}$: d is correct

Ans: b, d

59. The tension in the string is provided by the centripetal force hence $T_0 = \frac{mv_0^2}{r_0} = \frac{0.25 \times 4^2}{0.80} = 5\,N$

The initial kinetic energy of the circulating block $K_0 = \frac{1}{2} m v_0^2 = \frac{1}{2} \times 0.25 \times 4^2 = 2\,J$

When the string is pulled down, the radius decreases however the angular momentum is conserved as

$mvr = mv_0 r_0 \Rightarrow v = \frac{v_0 r_0}{r}$

The tension in the new position is $T = \frac{mv^2}{r} = \frac{mv_0^2 r_0^2}{r \times r^2} = \frac{mv_0^2}{r_0} \times \frac{r_0^3}{r^3} = \frac{T_0 \times r_0^3}{r^3}$; a is wrong

The kinetic energy in the new location is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_0\, r_0}{r}\right)^2 = K_0 \times \frac{r_0^2}{r^2}$; b is correct

The kinetic energy in the new location with $r_1 = \frac{r_0}{2}$ is $K = K_0 \times \left(\frac{r_0}{r_1}\right)^2 = K_0 \times \left(\frac{r_0}{\frac{r_0}{2}}\right)^2 = 4K_0$

Hence the work done by the tension T is = gain in kinetic energy W = 4 K_0 – K_0 = 3 K_0 ; d is wrong

For breaking the string, the tension must exceed the breaking strength so when $T = 40\,\text{N}$, we write $T = \frac{T_0 \times r_0^3}{r^3} \Rightarrow 40 = \frac{5\times 0.8^3}{r^3} \Rightarrow r^3 = \frac{5\times 0.8^3}{40} = \left(\frac{0.8}{2}\right)^3$ or $r = 0.40\ m$; c is correct

Ans: b, c

60. The change in flux through the circular coil when the magnetic field is suddenly reversed is

$\Delta\phi = NB\Delta S - (-NB\Delta S) = 2NB\Delta S = 2\times 2000\times 1\times 0.001 = 4.0\ weber$; b is correct

The induced charge in the circular copper coil is $q_{ind} = \frac{\text{change in flux}}{R} = \frac{2\,\text{NB}\,\Delta\,\text{S}}{R}$ ……. (1)

R is the total resistance of the circuit containing the circular coil

Let the charge passed through the Galvanometer turns the coil by θ then

$\frac{2NB\Delta S}{R} = \frac{T}{2\pi}\times\frac{c}{nAB_0}\times\theta$ …… (2)

The time period of torsional oscillations of the oscillating Galvanometer coil $T = 2\pi\sqrt{\frac{I}{c}}$

Or $T = 2\pi\sqrt{\frac{2.7\times 10^{-6}}{3\times 10^{-3}}} = 2\pi\times 3\times 10^{-2}\ \text{sec} = 0.1885 \cong 0.19\,S$; a is correct

The observed deflection (throw) of 40 mm on the linear scale at 1.0 m distance gives $\theta = \frac{1}{2}\times\frac{40}{1000}$ in eqn (2) the factor $\frac{1}{2}$ is used because of the fact that the deflection of the reflected ray is twice to the reflection of the plane mirror. Thereby

$$q_{ind} = \frac{\Delta\phi}{R} = \frac{2NB\Delta S}{R} = \frac{T}{2\pi}\times\frac{c}{nAB}\times\theta = \sqrt{\frac{I}{c}}\times\frac{c}{nAB}\times\frac{40}{1000}\times\frac{1}{2}$$

$$q_{ind} = \frac{2\times 2000\times 0.001\times 1}{R} = \sqrt{\frac{2.7\times 10^{-6}}{3\times 10^{-3}}}\times\frac{3\times 10^{3}}{100\times 15\times 10^{-4}\times 0.1}\times 0.02 = \frac{6}{5}\times 10^{-4} = 120\,\mu C$$

Thus $q_{ind} = 120\,\mu C$ and not 240 μC c is wrong

Further $\frac{4}{R} = 120\times 10^{-6} \Rightarrow R \cong 33.3\ k\Omega$ d is correct

Ans: a, b, d