INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY 2014

Q. Paper Code: A 412

1) A certain star appears to rise from the east at 9:30 pm on a particular day. From the same place of observation, after 25 days the same star appears to rise from east at about

   a) 11:10 pm b) 7:50 pm c) 8:05 pm d) 10:20 pm

Solution (a) : Every day rise of star is delayed by 4 min due to revolution of earth around the sun. That is 25X4=100min=1hr 40 min.

2) A mixture of two moles of hydrogen and one mole of argon gas is taken in a closed container at room temperature. Consider the following two statements

   i) The average kinetic energy of each atom of H and Ar are the same.
   ii) The pressure due to argon gas is more than that due to hydrogen gas.

   a) Both statement i) and ii) are correct
   b) Statement i) is correct while statement ii) is incorrect
   c) Both statement i) and ii) are incorrect
   d) Statement i) is incorrect while statement ii) is correct

Solution (b): The kinetic energy depends on the temperature of the gas. Since both the gasses are at same temperature, its kinetic energy must be same.
The pressure due to the mixture of gasses is given by $P = (n_1 + n_2)K_B T$. That is the pressure depends on temperature, number of moles of the gas present and not on the atomic mass. In this case pressure due to H is more than Ar.

3) If $f(x) = \sqrt{x^2 - 3x + 2} + \dfrac{1}{\sqrt{x^2 - 3x - 4}}$, the domain of f(x) is

   a) $(-\infty, -1) \cup (4, \infty)$ b) $[-\infty, -1] \cup [4, \infty]$

   c) $(-\infty, 4)$ d) None of these

Solution (a) $\sqrt{x^2 - 3x} + 2 \geq 0 \Rightarrow$ Domain is $(-\infty, -1] \cup (4, \infty)$

$\sqrt{x^2 - 3x} - 4 > 0 \Rightarrow$ Domain is $(-\infty, -1) \cup (4, \infty)$

$\{(-\infty, -1] \cup [2, \infty)\} \cap \{(-\infty, -1) \cup (4, \infty)\}$

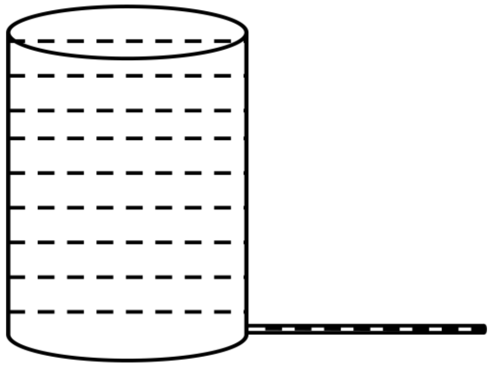
$\Rightarrow (-\infty, -1) \cup (4, \infty)$

4) Order of magnitude calculation in astronomy plays an important role in understanding the concepts. The order of magnitude of number of Fe atom present in a solid cube of volume 1 cc is (molecular mass of iron is around 56 $gmol^{-1}$)

   a) 10^{25} b) 10^{27} c) 10^{22} d) 10^{21}

Solution(c): Density of materials is of the order of gcc^{-1}. Hence the mass of 1cc of Fe = 1X1cc=1g

The number of atoms present = (1/56) X 6.02 X $10^{23} \approx 10^{22}$



5) The viscosity of water can be determined by studying the Poiseuille's flow in a capillary tube connected to cylindrical container as shown in the adjacent figure. The density and viscosity of water are given by 1.00 gcc^{-1} and 1.00 $mN\text{-}sm^{-2}$. The radius and length of the capillary is 1.00 mm and 20.0 cm. If Radius of the vessel is 15.0cm , then the time taken for height of water to reduce to half the initial is

a)15.02 minute b) 1.502 hour

c) 22.03 minute d) 42.43 minute

Solution (d): The rate of flow of water is given by $\frac{dV}{dt} = \frac{1}{8\eta}\frac{(P_2-P_1)}{l}\pi r^4 = \frac{1}{8\eta}\frac{h\rho g}{l}\pi r^4$

Here h is the height of the water column above the capillary tube.

The rate of flow can be written as $\frac{dV}{dt} = A_{Cy}(-\frac{dh}{dt})$. Here A_{Cy} is the area of the cylindrical vessel

Thus we have $A_{Cy}\frac{dh}{dt} = \frac{1}{8\eta}\frac{h\rho g}{l}\pi r^4$

Or $-\frac{dh}{dt} = \frac{1}{8\eta}\frac{\rho g \pi r^4}{l\ A_{Cy}}h.$

Or $\frac{dh}{dt} = -\lambda h.$ This is first order differential equation. The solution is given by

$$h = h_0 e^{-\lambda t}$$

That is $t = \frac{1}{\lambda} ln\frac{h_0}{h} = \frac{ln2}{\lambda}$

The value of λ is given by $\lambda = \frac{1}{8\eta}\frac{\rho g \pi r^4}{l\ A_{Cy}} = \frac{1000X9.8X(10^{-3})^4}{8X1X10^{-3}X0.2X(0.15)^2} = 2.72X10^{-4}s^{-1}$

Thus $t = \frac{0.693}{2.72X10^{-4}} = 2545.7s = 42.43\ minute$

6) The range of the function, x ∈ R (square bracket indicates integer values only)

$f(x) = \frac{\sin(\pi[x^2+1])}{x^4+1}$ is

a) [0, 1] b) [–1, 1] c) {0} d) None of these

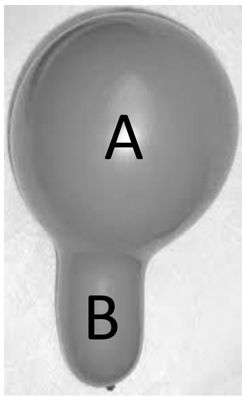
Solution (c): Since $[x^2 + 1]$ is an integer ∀ x ∈ R

∴ $\sin(\pi\,[x^2 + 1]) = 0$ ∀ x ∈ R

∴ Range is {0}

7) An inflated balloon is shown in the adjacent figure. Two regions of interest are marked on it as A and B. Which of the following statement is incorrect (R_A and R_B are radii of curvature at A and B respectively)

a) The pressure at A and B are the same.
b) The pressure at A and B are different.
c) Tension of the rubber membrane at A (T_A) and B (T_B) are different.
d) The ratio T_A/R_A at A and T_B/R_B at B are same.



Solution (b): The pressure at A and B has to be same, else air at A would flow to B. The tension at A and B are different since stretching is different. The ratio $T/_R$ is equal to pressure difference between the outside and inside the balloon.

8) A meteor shower occurs when
   a) Earth passes through the asteroid belt
   b) Earth passes through a swarm of dust particles in space, which are remnants of a comet
   c) Head of a comet hits the earth's atmosphere
   d) A distant star explodes

   Solution (b)

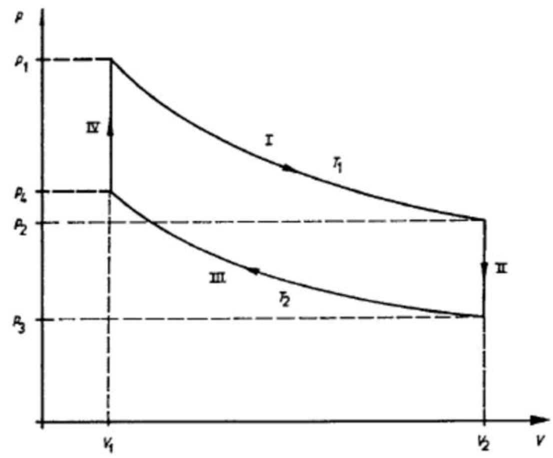
9) If tan 25° = x, then $\frac{\tan 155° - \tan 115°}{1+\tan 155° \tan 115°}$ is equal to

   a) $\frac{1-x^2}{2x}$ b) $\frac{1+x^2}{2x}$ c) $\frac{1+x^2}{1-x^2}$ d) $\frac{1-x^2}{1+x^2}$

   Solution (a): tan 155° = tan (180° – 25°)
   tan 115° = tan (90° + 25°)

$$\frac{\tan 155° - \tan 115°}{1+\tan 155° \tan 115°} = -\frac{\tan 25° + \cot 25°}{1+\tan 25° \cot 25°} = \frac{-x+\frac{1}{x}}{2} = \frac{1-x^2}{2x}$$

10) Sterling engine involves cyclic process similar to Carnot cycle. It consists of two isothermal process at temperature T_1 and T_2. Adiabatic processes are practically difficult to achieve and hence they are replaced by isochoric processes as shown in the adjacent figure. The efficiency of the engine is equal to (γ is the ratio of C_p to C_V)



   a) $\frac{1}{\gamma}\frac{T_1-T_2}{T_1}$
   b) $\frac{T_1-T_2}{T_1}$
   c) $\frac{1}{(\gamma-1)}\frac{T_1-T_2}{T_1}$
   d) $\frac{1}{2\gamma}\frac{T_1-T_2}{T_1}$

   Solution (b) : Efficiency is ratio of work done by the system to energy supplied to the system. Comparing with Carnot's engine, the only difference is the isochoric processes in place of adiabatic processes. The isochoric process neither contributes to work done nor supplying energy to system like adiabatic process hence the expression for efficiency is same.

11) Sun appears against different constellation in different months of an year due to
   a) Earth's rotation about its axis
   b) Earth's orbit motion around the sun
   c) Sun's motion around the centre of milky way galaxy
   d) Precession of earth's axis of rotation or precession of equinoxes.

   Solution (b)

12) Number of solutions of the equation tan x + sec x = 2 cos x lying in the interval [0, 2π] is

a) 0 b) 1 c) 2 d) 3

Solution (c): $\frac{\sin x+1}{\cos x}=2\cos x$

sin x + 1 = 2 $\cos^2$ x

sin x + 1 = 2(1 – $\sin^2$ x)

2 $\sin^2$ x + sin x – 1 = 0

(2 sin x – 1) (sin x + 1) = 0 $\Rightarrow \sin x=\frac{1}{2}$ ($\because$ sin x + 1 $\neq$ 0 since cos x =0 implies the equation become indeterminant)

$x=\frac{\pi}{6}, \frac{5\pi}{6}$ in [0, 2π]

13) An experiment is done to measure the Brewester's angle and hence refractive index of a glass slab. The angle obtained is $58.0^0 \pm 0.1^0$. The refractive index of the glass is given by

a) 1.60± 0.356 b) 1.60 ± 0.06 c) 1.60± 0.6 d)1.60± 0.006

Solution (d): The refractive index is $n = tan\theta = 1.60$ and

$\Delta n = sec^2\theta\Delta\theta = 3.56X0.1\ X\left(\frac{\pi}{180}\right) = 0.0062$

14) The primary purpose of using a parabolic mirror in a reflecting telescope is

a) To collect more light from a distant galaxy
b) To increase the magnification
c) To overcome diffraction effects
d) To correct for spherical aberration

Solution (d)

15) If P(n) = $n^3 + (n + 1)^3 + (n + 2)^3$ where n $\in$ N, then P(n) is divisible by

a) 9 b) 7 c) 13 d) 15

Solution (a):

When n = 1, 1 + 8 + 27 = 36 – rules out b) and d)

When n = 2, 8+27+64=99 – rules out c)

16) An aluminium sphere of radius R is heated from 20^0 to 100^0. If the coefficient of volume expansion of Al is $6.9X10^{-5}K^{-1}$ then the percentage change in the moment of inertia of the sphere is

a) 0.368% b) 1.1% c) 0.245% d) 0.876%

Solution (a): The ratio of change in radius, $\frac{\Delta R}{R} = \frac{\gamma}{3}\Delta t$. The ratio of change in moment of inertia is $\frac{\Delta I}{I} = 2\frac{\Delta R}{R} = 2\frac{\gamma}{3}\Delta t = 2X\frac{6.9X10^{-5}}{3}X80 = 0.00368 = 0.368\%$

17) The correct sequence of the objects in the ascending order of distance from the sun, is

a) Kupier belt, Uranus, asteroid belt and oort cloud
b) Uranus, asteroid belt, oort cloud and Kupier belt

c) Oort cloud, asteroid belt, Uranus and Kupier belt
d) asteroid belt, Uranus , Kupier belt, and Oort cloud

Solution: (d)

18) Sum to 7 terms of $1.3^2 + 3.5^2 + 5.7^2 + \ldots$is

a) 6769 b) 6760 c) 6768 d) 6770

Solution (a): $t_n = (2n - 1)(2n + 1)^2 = (2n - 1)(4n^2 + 1 + 4n)$

$s_n = \Sigma(2n - 1)(2n + 1)^2 = 8\Sigma n^3 + 4\Sigma n^2 - 2\Sigma n - \Sigma 1$

$s_7 = 8\Sigma 7^3 - 2\Sigma 7 + 4\Sigma 7^2 - \Sigma 1$

$$8\frac{8^2 7^2}{4} - 2\frac{7\times 8}{2} + 4\frac{7\times 8\times 15}{6} - 7$$

= 6769

19) Two particles A and B of equal mass are at rest at positions (2m,3m) and (4m,-1m) respectively. They start moving with velocities $(\hat{\imath} + 2\hat{\jmath})$ ms^{-1} and $(-2\hat{\imath} + \hat{\jmath})$ ms^{-1} respectively. At the end of 3 s, the position of their centre of mass will be

a) (1.5m, 5.5m) b) (3.0m, 1.0m) c) (-3.0m, 9.0m) d) (4.5m, 3.0m)

Solution (a): The position of a particle at the end of 3 s is $\vec{R} = \vec{R}_0 + \vec{v}t$

After 3 s, $R_A = (2\hat{\imath} + 3\hat{\jmath}) + 3(\hat{\imath} + 2\hat{\jmath}) = 5\hat{\imath} + 9\hat{\jmath}$ and

$$R_B = (4\hat{\imath} - 1\hat{\jmath}) + 3(-2\hat{\imath} + \hat{\jmath}) = -2\hat{\imath} + 2\hat{\jmath}$$

The co-ordinates of centre of mass, $x = \frac{5-2}{2} = 1.5$, $y = \frac{9+2}{2} = 5.5m$

20) Among the planets

(i) Mercury (ii) Mars (iii) Venus (iv)Earth (v) Jupiter

The terrestrial planets are

a)(ii) and (iv) only b) (ii), (iii), (iv) and (v) only
c)(i), (iii) and (iv) only d) (i), (ii), (iii) and (iv) only

Solution (d)

21) If $(x + iy)^{1/3} = (a + ib)$, then $\frac{x}{a} + \frac{y}{b}$ equals

a) $4(a^2 - b^2)$ b) $2(a^2 - b^2)$ c) $2(a^2 + b^2)$ d) none of these

Solution (a): $(x + iy) = (a + ib)^3 = a^3 + i3ab(a + ib) + (ib)^3$

$= a^3 + i3a^2b + 3ab^2i^2 + i^3b^3$

$= a^3 - 3ab^2 + i(3a^2b - b^3)$

$\Rightarrow x = a^3 - 3ab^2$ $\quad y = 3a^2b - b^3$

$\frac{x}{a} = a^2 - 3b^2$ $\quad \frac{y}{b} = 3a^2 - b^2$

$$\therefore \frac{x}{a} + \frac{y}{b} = 4a^2 - 4b^2$$

22) A spherical shell of mass M and radius R filled completely with a liquid of same mass and set to rotate about a vertical axis through its centre has a moment of inertia I_1 about the axis. The liquid starts leaking out of the hole at the bottom. If moment of inertia of the system is I_2 when the shell is half filled and I_3 is the moment of inertia when entire water drained off, then

a) $\frac{I_1}{I_2} \approx 1.5$ b) $\frac{I_1}{I_2} \approx 0.67$ c) $\frac{I_1}{I_3} \approx 1.6$ d) $\frac{I_2}{I_3} \approx 1.4$

Solution (c) Moment of inertia of a shell, $I_3 = \frac{2}{3}MR^2$

Moment of inertia of a completely filled sphere, $I_1 = \frac{2}{3}MR^2 + \frac{2}{5}MR^2 = \frac{16}{15}MR^2$

Moment of inertia of a half filled sphere, $I_2 = \frac{2}{3}MR^2 + \frac{2}{5}\frac{M}{2}R^2 = \frac{26}{30}MR^2$

23) The planet in which sun appears to rise in the west is

a) Venus b) Uranus c) Saturn d) Mercury

Solution: a) Venus rotates from east to west, opposite to that on earth

24) If x = 2 + 5i, then the value of $x^3 - 5x^2 + 33x - 19$ is equal to

a) -5 b) -7 c) 7 d) 10

Solution (d): x = 2 + 5i

$(x - 2)^2 = 25i^2$

$x^2 - 4x + 29 = 0$

$x - 2 = 5i$

$x^2 + 4 - 4x = -25$

$$
\begin{array}{r|l}
 & x - 1 \\
\hline
x^2 - 4x + 29 & x^3 - 5x^2 + 33x - 19 \\
 & x^3 - 4x^2 + 29x \\
 & - \quad + \quad - \\
\hline
 & -x^2 + 4x - 19 \\
 & -x^2 + 4x - 29 \\
 & + \quad - \quad + \\
\hline
 & 10
\end{array}
$$

25) The lift force per unit length of wingspan (F) of an aircraft depends on width L of the its wing, velocity v, air density ρ. The correct expression for F is (Where k is a dimensionless constant).

(a) $F = kLv^2\rho$ (b) $F = kv^3\rho$ (c) $F = kL^2v^2\rho^2$ (d) $F = kLv\rho^2$

Solution (a) Dimensional analysis;

$$\frac{Force}{length} = L^a v^b \rho^c$$

$$\frac{MLT^{-2}}{L} = L^a(LT^{-1})^b(ML^{-3})^c$$

$$ML^0T^{-2} = M^c L^{a+b-3c}T^{-b}$$

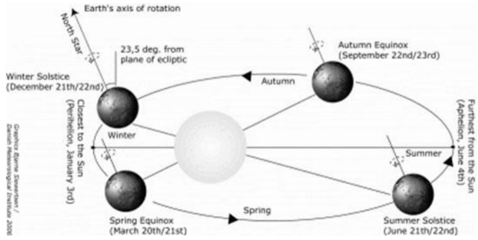
c=1; b=2 and a =1

26) For places equidistant from the equator in northern and southern hemisphere choose the correct statement,

a) Summers are warmer and winters are colder in the southern hemisphere
b) Summers are colder and winters are warmer in the southern hemisphere

c) Both summers and winters are warmer in northern hemispheres
d) Both summers and winters are colder in northern hemisphere

Solution (a): In southern hemisphere summer occurs during December-January, when earth is nearest to sun, the winter occurs during June when earth is farthest to sun.



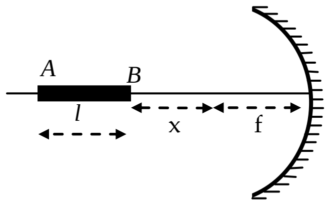
27) The absolute temperature T of a gas is plotted against its pressure P for two different constant volumes V_1 and V_2 where $V_1 > V_2 \cdot T$ is plotted along $x-$axis and P along $y-$axis.
   a) Slope for curve corresponding to volume V_1 is greater than that corresponding to volume V_2
   b) Slope for curve corresponding to volume V_2 is greater than that corresponding to volume V_1
   c) Slope for both curves are equal
   d) Slope for both curves are unequal such that they intersect at $T = 0$

Solution (b): P=nRT/V. Slope = nR/V; as V increases slope decreases.

28) The phenomenon of streams of hot hydrogen gas bursting out of the sun's surface for a height and falling back on the surface are called
   a) Solar flares b) spicules c) solar prominences d) solar storms

Solution (c)

29) A rod of length l = 10 cm and diameter d = 2.0 mm is placed along the axis of a concave mirror of focal length f = 10 cm as shown in the figure. End B is at distance x = 10 cm from the focus F of the mirror. Then
   a) Length of the image is 5.0 cm and end A is 1.0 mm thick
   b) Length of the image is 6.0 cm and end A is 1.0 mm thick
   c) Length of the image is 5.0 cm and end A is 2.0 mm thick
   d) Length of the image is 6.0 cm and end A is 4.0 mm thick



Answer (a)
Solution:
The image of B is B itself since $U_B = x + f = 20\ cm = R$ for end A,
$U_A = l + x + f = 30\ cm$
Its image distance is $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30}$
$\Rightarrow$ v = 15 cm
so the image length = x + f – v = 5.0 cm
lateral magnification of B in 1.0
and that of A is $\frac{15}{30} = \frac{1}{2} = \frac{{r_A}^1}{r_A} = {r_A}^1 = 1.0\ mm$
so, option (a) is true

30) Read the following statements
   (i) Photons emitted by the sun are produced by electromagnetic processes in the photosphere

(ii) A photon liberated in the core of the sun, takes about a few hundred thousand years to reach its surface

Pick out the correct option given below

a) (i) is true but (ii) is wrong
b) Both (i) and (ii) are wrong
c) (i) is wrong but (ii) is true
d) Both (i) and (ii) are true

Solution (c)

31) The remainder obtained when $31 \times 32 \times 33 \times 34 \times 35 \times 36$ is divided by 29 is

a) 2 b) 3 c) 15 d) 23

Solution (d):2X3X4X5X6X7 (mod 29)

2X3(−1) (1) (mod 29)

−6(mod 29)

$\Rightarrow$ 23 (mod 29)

32) Out of the celestial objects, redgaint (RG), white dwarfs (WD), neutron star (NS) and black holes (BH), gravitational contraction is countered by degeneracy pressure in

a) RG, WD and NS only b) WD and NS only c) WD only d) WD, NS and BH only

Solution (b): There is no nuclear fusion in WD, NS and BH. In BH there is nothing to counter gravitational contraction. In WD and NS the stability is due to degeneracy pressure (a result of Pauli's exclusion principle) due to electrons and neutrons respectively.

33) For spherical mirrors which of the following depend on whether the rays are paraxial or not?

a) Pole
b) Focus
c) Radius of curvature
d) Principal axis

Answer (b)

Solution:

The rays close and parallel to the principal axis are focused at the geometric focus whereas those farther away are brought to focus at a point nearer to the pole. It is the focus that determines whether the rays are paraxial or not.

34) The number of real roots of $(7+4\sqrt{3})^{|x|-8}+(7-4\sqrt{3})^{|x|-8}=14$ is

a) 0 b) 2 c) 3 d) 4

Solution (d): $7-4\sqrt{3}=\dfrac{1}{7+4\sqrt{3}}$

If $a=(7+4\sqrt{3})^{|x|-8}$ then $a+\dfrac{1}{a}=14$

$$a^2-14a+1=0$$

$$a=\frac{14\pm\sqrt{194-4}}{2}=\frac{14\pm 8\sqrt{3}}{2}=7\pm 4\sqrt{3}$$

$\therefore\ (7+4\sqrt{3})^{|x|-8} = (7+4\sqrt{3})^{\pm 1}$

$\therefore$ |x| – 8 = ± 1 or |x| = 9,

$\therefore$ x = ±9 or ±7

$\therefore$ 4 roots

35) Ptolemy developed the concept of epicycles in planetary motion mainly to
   a) Assert that earth is the centre of the universe.
   b) Show that all planets revolve around the sun
   c) Explain the observed retrograde motion of planets
   d) To prove that circular motion is a natural state of motion for all celestial objects.

Solution (c): The apparent motion of a planet in the opposite direction for sometime as against their generally forward motion as observed from sun is called retrograde motion.

36) In Indian astronomy, the Zodiacal belt on the celestial sphere is divided into constellations. Each constellation is further divided into smaller units called Nakshatras. The mean time taken by sun to pass across a Nakshatra on the celestial sphere is about
   a) 13.5 days   b) 27.3 days   c) 23 hrs 56 minutes   d) 30 days

Solution: (a) There are 27 nakshatras spread over 360^0. Hence time required to pass each nakshatra is 360/27=13.5 days

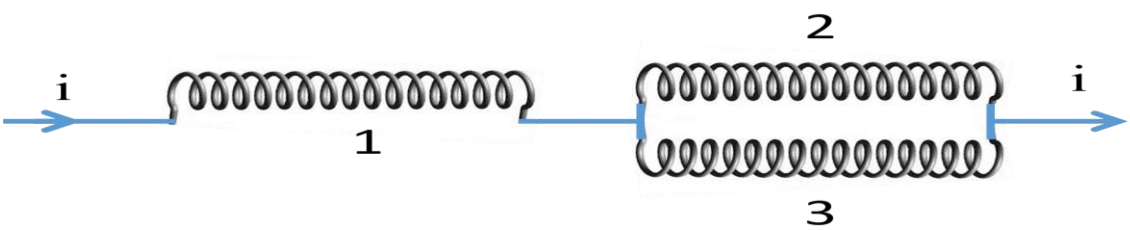
37) The value of $\lim\limits_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{2n}\right]$ is

   a) 1   b) $\log_e 2$ c) $\frac{1}{3}$ d) $\frac{\pi}{4}$

Solution (d): $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2+r^2} = \lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\frac{1}{1+\frac{r^2}{n^2}}$

$$= \int_0^1 \frac{1}{1+x^2}dx = \tan^{-1}x\Big]_0^1 = \frac{\pi}{4}$$

38) Three closely wound coils identical in respect of number of turns material of wire and the radius, are connected as shown in the figure.



The magnetic fields due to the steady current in coils 1, 2 and 3 are respectively $\overrightarrow{B_1}, \overrightarrow{B_2}\ and\ \overrightarrow{B_3}$. Then

   a) $\frac{\overrightarrow{B_1}}{2} = \overrightarrow{B_2} = \overrightarrow{B_3}$ b) $|B_2| = |B_3| = |\frac{B_1}{2}|$

   c) $\overrightarrow{B_2} = \overrightarrow{B_3}$ d) $|B_2| \neq |B_3|$

Solution(b): The direction of B in coil 3 is opposite since windings in the coil in an opposite sense. Since the current coil in 2 and 3 is half that in coil 1, magnitude of B in 2 and 3 is half that in coil 1. Thus option (b) is correct.

Passage question (39 to 42): Read the following passage and answer the questions below

The magnitude scale of the brightness of stars is logarithmic. As magnitude increases by 5, brightness decreases by factor of 100. The magnitudes of stars as observed from earth are called apparent magnitudes (m). Magnitude of a star as observed from a standard distance of 10 parsec is called the real or true magnitude (M). The relation between m and M may be written as

$$M = m + 5 - 5logd$$

Where d is the distance from earth to the star in the units of parsec. One parsec = 3.25 Ly

39) The brightness of two stars is b_1 and b_2. If they have apparent magnitude m_1 and m_2 then the correct relation is

a) $m_1 - m_2 = 5\log\left(\frac{b_1}{b_2}\right)$ b) $m_1 - m_2 = 5\log\left(\frac{b_2}{b_1}\right)$

c) $m_1 - m_2 = 2.5\log\left(\frac{b_1}{b_2}\right)$ d) $m_1 - m_2 = 2.5\log\left(\frac{b_2}{b_1}\right)$

Solution (d): Since the scale is logarithmic and brightness increasing with decrease in magnitude we can write

$$klogb = -m$$

Or

$$k(logb_1 - logb_2) = -m_1 - (-m_2)$$

$$klog\left(\frac{b_1}{b_2}\right) = m_2 - m_1$$

Brightness ratio of two stars of magnitude 6 and 1 is given by 100. That is

$$klog(100) = 6 - 1$$

Thus k = 2.5

40) The star 'Jyeshta' has an apparent magnitude of about 0.92 and the real magnitude of about (-5.1). The distance of the star from earth is about

a) 250Ly b) 160Ly c) 520Ly d) 210Ly

Solution (c): $log(d) = \frac{-M+m+5}{5} = \frac{5.1+0.92+5}{5}$

d=160 parsec and 520Ly

41) Sirius is a binary star with the star Sirius A having m = -1.46 and M = 1.42 and Sirius B having m=8.30 and M=11.18. As seen from the earth the star A is P times brighter than B. Then P is about

a) 250 b) 3900 c) 8017 d) 10,000

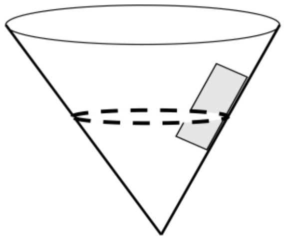
Solution (c) $\frac{b_A}{b_B} = 10^{\frac{m_B-m_A}{2.5}} = 10^{\frac{8.3-(-1.46)}{2.5}} = 8017$

42) Two stars A and B have apparent magnitudes of -1.2 and 0.8 respectively, but their real magnitudes are same and equal to -2.4. Then we may conclude

a) Both A and B have same surface temperature
b) Both A and B are at the same distance from earth
c) As observed from the earth B is nearer to earth
d) Star A appears to be brighter than B by a factor of 6.3

Solution (d): $\frac{b_A}{b_B} = 10^{\frac{m_B-m_A}{2.5}} = 10^{\frac{0.8-(-1.2)}{2.5}} = 6.31$

43) Consider a block moving in a horizontal circular path on the inner surface of a cone. The normal force on the block is



a)mg b) always greater than mg

c) may be greater or less than mg d) always less than mg

Solution (b): b is correct. Apart from force mg, centrifugal force also presses the block to the inner surface.

44) Three objects, all of mass M, are released simultaneously from the top of an inclined plane of inclination 60^0 and height H. The objects are described as follows

i) a cube of side R.

ii) a solid cylinder of radius R

iii) a hollow cylinder of radius R

Assume the cylinders roll down the plane without slipping and the cube slides down the plane (coefficient of friction 0.4). Which object(s) reach(s) the bottom of the plane first?

a) I b) II c) III d) I & II

Solution (a) acceleration of cube $a_1 = gsin\theta - \mu gcos\theta = 0.66g$

Acceleration of solid cylinder $a_2 = \frac{mgsin\theta}{m+\frac{I}{r^2}} = \frac{mgsin\theta}{m+\frac{\frac{1}{2}mr^2}{r^2}} = \frac{2}{3}gsin\theta$ =0.57g

Acceleration of hollow cylinder $a_3 = \frac{1}{2}gsin\theta$=0.433g

45) The value of $\int_0^{\frac{\pi}{4}} e^{\sqrt{x}} dx$

a) e^2 b) e c) 2e d) $2e^2$

Solution (d): $\int_1^4 e^{\sqrt{x}}\, dx = 2\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}} \sqrt{x} dx$; take $z = \sqrt{x}$

$= 2\int_1^2 ze^z dz = 2[ze^z - e^z\Big]_1^2$ $\frac{1}{2\sqrt{x}} dx = dz$

= $2[2e^2 - e^2 - 0]$

= $2e^2$

46) If $x + \frac{1}{x} = 2\cos\theta$ and $y + \frac{1}{y} = 2\cos\phi$ then $\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}}$ is

a) $2\cos\left(\frac{\theta - \phi}{2}\right)$ b) $2\cos\left(\frac{\theta + \phi}{2}\right)$ c) $2\sin\left(\frac{\theta + \phi}{2}\right)$ d) $2i\sin\left(\frac{\theta - \phi}{2}\right)$

Solution(d): $x+\frac{1}{x}=2\cos\theta \Rightarrow \cos\theta = x$ *and* $y+\frac{1}{y}=2\cos\phi \Rightarrow y=\cos\phi$

$$\sqrt{\frac{x}{y}}=\left(\frac{\cos\theta}{\cos\phi}\right)^{\frac{1}{2}}=[\cos(\theta-\phi)]^{\frac{1}{2}}=\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\sqrt{\frac{x}{y}}=\cos\left(\frac{\theta-\phi}{2}\right)+i\sin\left(\frac{\theta-\phi}{2}\right)$$

$$\sqrt{\frac{y}{x}}=\cos\left(\frac{\theta-\phi}{2}\right)-i\sin\left(\frac{\theta-\phi}{2}\right)$$

$$\sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}=2i\sin\left(\frac{\theta-\phi}{2}\right)$$

Linked questions 47 to 51: Sun

47) Sun is the closest star and main source of energy for all the living beings on the earth. For all practical purpose it can be assumed to be a black body radiating energy in all wavelengths. The maximum intensity of radiation occurs at a wavelength of 500 nm. The temperature of sun is (Wien's constant = $2.8X10^{-3}$ mK)

a) 5600K b) 14000K c) 17800K d) 4800K

Solution (a): From Wien's law; $T=\frac{\sigma}{\lambda_{max}}=\frac{2.8X10^{-3}}{500X10^{-9}}=5600K$

48) Once the temperature of Sun is known, the Luminosity which is nothing but the total energy emitted per second can be determined. If the radius of Sun is $6.95X10^{8}$ m, then the luminosity of sun is (Stefan's constant = $5.67X10^{-8}$ $Wm^{-2}K^{-4}$)

a) $4.2X10^{24}$W b) $3.38X10^{26}$W c) $3.38X10^{24}$W d) $4.2X10^{26}$W

Solution (b): From Stefan's law, Power of emitted radiation , $L=4\pi R^2\sigma T^4 = 4\pi X(6.95X10^8)^2X5.67X10^{-8}X(5600)^4$ =$3.38X10^{26}$W

49) The energy that comes out of the Sun is mainly due to thermo nuclear fusion that converts hydrogen into helium through Proton-Proton cycle. The reaction given below produces an amount of energy equal to 26.7MeV.

$$4\,{}_1H^1 \rightarrow {}_2He^4+2e^+ +2\nu+26.7\,Mev$$

Assuming that all the energy produced in the fusion reaction is emitted, the number of reactions that has to occur in one second is given by

a) $1.26X10^{19}$ b) $1.26X10^{26}$ c) $7.91X10^{37}$ d) $1.08X10^{42}$

Solution(c): The number of reaction per second,

$$n=\frac{amount\ of\ energy\ emitted\ by\ sun\ in\ one\ second}{amount\ of\ energy\ emitted\ in\ one\ reaction}=\frac{3.38X10^{26}}{26.7X1.6X10^{-13}}=7.91X10^{37}$$

50) The energy released in a nuclear reaction is due to conversion of some amount of mass. Calculate the amount of hydrogen converted to helium in one second due to the fusion reaction (mass of proton =$1.67X10^{-27}$kg)

a) 528.38 million ton b) 345.14 million ton c) 356.15 million ton d) 768.34 million ton

Solution (a): Mass of hydrogen converted,

$$m = 7.91X10^{37}X4X1.67X10^{-27} = 5.2838X10^{11}Kg$$

51) A star is said to be in its prime age until all hydrogen in the core converts to helium. Most part of its life is spent in this phase. Assuming only 10% of total mass of sun is in the core and 74% of sun is made of hydrogen calculate the approximate life time of sun (mass of sun = $2X10^{30}$kg)

a) 12 billion years b) 9.1 billion years c) 9.8 billion years d) 8.9 billion years

Solution (d) : Life time, $t = \frac{2X10^{30}X0.74X0.1}{5.2838X10^{11}} = 2.8X10^{17}s = 8.9\ billion\ years$

52) If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, $|\vec{a}| = 2;$ $|\vec{b}| = 3,$ $|\vec{c}| = 4$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$ then $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is

a) 24 b) 12 c) $12\sqrt{3}$ d) $24\sqrt{3}$

Solution(c): $|\vec{b} \times \vec{c}| = bc\sin\frac{2\pi}{3} = 12\frac{\sqrt{3}}{2} = 6\sqrt{3}$

$$[\vec{a}\vec{b}\vec{c}] = \vec{a}(\vec{b} \times \vec{c}) = |\vec{a}||\vec{b} \times \vec{c}|\cos 0$$

$$= 2 \times 6\sqrt{3} = 12\sqrt{3}$$

53) If $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors and $\hat{i} \times \hat{j} = \hat{k}$, then $(\hat{i} + \hat{j}) \times (\hat{j} - \hat{i})$ is

a) $\hat{k}$ b) $2\hat{k}$ c) $-\hat{k}$ d) $-2\hat{k}$

Solution(b): $(\hat{i} + \hat{j}) \times (\vec{j} - \vec{i}) = \hat{i} \times \hat{j} + \hat{j} \times \hat{j} - \hat{i} \times \hat{i} - \hat{j} \times \hat{i}$

$$= 2\hat{k}$$

54) If $\log_5 2$, $\log_5 (2^x - 5)$ and $\log_5\left(2^x - \frac{7}{2}\right)$ are in A.P, then x is equal to

a) $\frac{1}{2}, \frac{3}{2}$ b) 3 c) 4, 5 d) 8

Solution(b): $2\log_5(2^x - 5) = \log_5 2 + \log_5\left(2x - \frac{7}{2}\right)$

$$(2^x - 5)^2 = 2\left(2^x - \frac{7}{2}\right)$$

$$(2^x)^2 + 25 - 10(2^x) = 2.2^x - 7$$

$(2^x)^2 - 12.2^x + 32 = 0$ $y^2 - 12y + 32 = 0$

$(2^x)^2 - 12(2^x) + 32 = 0$ $(y - 8)(y - 4) = 0$

$2^x = 2^3;$ $2^x = 2^2$

x = 3; 2 since $x \neq 2$ (since $\log_5 (2^x - 5)$ is undefined for a negative number)

$\therefore$ x = 3

55) A loudspeaker produces sound by means of oscillations of a diaphragm whose amplitude is limited to 0.2µm. The minimum frequency above which dust particles sitting on the diaphragm loses the contact is

a) 352.5M Hz b) 1.114 KHz c) 7.80×10^{6}Hz d) 352.5Hz

Solution (b): The dust particle will lose contact with diaphragm when normal force on it is zero. The normal force will be $N = mg - m\omega^2 x$. ω is the angular frequency. For minimum frequency, the displacement (x) has to be maximum. Thus

$$0 = mg - m4\pi^2 f^2 A.$$

Or $f = \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2\pi}\sqrt{\frac{9.8}{2X10^{-7}}} = 1114.1\ Hz$

56) A block of mass 2kg connected to a spring of spring constant $8Nm^{-1}$ is allowed to oscillate on a rough horizontal surface. If the system experiences a $damping\ force = 0.230\ Xvelocity$, then the time required for the amplitude of resulting oscillation to fall to half of its initial value is

a) 0.693 s b) 12 s c) 0.08 s d) 14.3 s

Solution (b): The amplitude at time t is given by $A = A_0 e^{-\frac{bt}{2m}}$

Thus $t = \frac{2m}{b} ln\left(\frac{A_0}{A}\right) = \frac{2X2}{0.230}\ln(2) = 12.05s$

57) Two strings of a piano are identical in length, composition and diameter. One of the strings is tuned correctly to 300Hz. When the two strings are sounded together 2 beats per second is heard. The Percentage change in tension required to tune the string to match the frequency of the other string is

a) 1% b) 1.66% c) 1.33% d) 2%

Solution (c): The frequency of a string $f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$

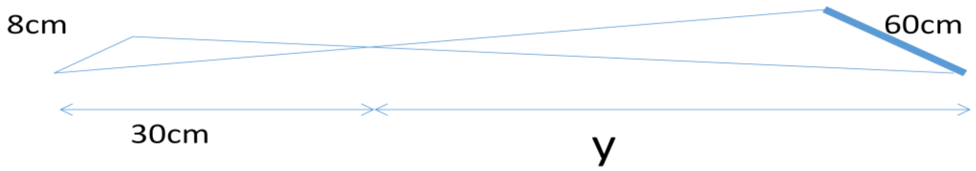
$$\frac{\Delta T}{T} = 2\frac{\Delta f}{f} = 2X\frac{2}{300} = 0.013$$

58) In an experiment to estimate the distance of a black board from a student using parallax method, 10 markings are made separated by 10.0cm from each as shown in the figure below. The student holds a pen in the hand in front of the eyes in the direction of the board. When she sees the pen with only left eye, it coincides with 8th marking on the board. When she sees the pen only with right eye, then the pen appears to coincide with 2nd marking. If the separation between eyes for normal human being is 8.0 cm and pen to eye distance is 30.0 cm, then distance from the student to the board is

a) 225cm b) 145cm c)255cm d)175cm

Solution (c): From the similar triangles shown in the figure: $\frac{Y}{30} = \frac{60}{8} \quad or\ y = 225cm$

Thus the distance to the board is 225+30=255cm.



59) In the expansion of $(1 + px)^n$, $n \in N$ the coefficient of x and x^2 are 8 and 24 respectively, then

a) n = 3, p = 2 b) n = 4, p = 2 c) n = 4, p = 3 d) n = 5, p = 3

Solution (b): $1 + {}^nC_1 px + {}^nC_2 (px)^2 + \ldots$

$$1 + npx + \frac{n(n-1)}{2} p^2 x^2$$

$\therefore$ np = 8 $\qquad \frac{n(n-1)}{2} p^2 = 24$

$p = \frac{8}{n} \qquad \frac{n(n-1)}{2} \times \frac{64}{n^2} = 24$

$$\frac{(n-1)}{n} = \frac{24}{32} = \frac{3}{4}$$

4n – 4 = 3n

n = 4 and p = 2

60) The value of $1 + \frac{2}{5} + \frac{3}{25} + \ldots$ to ∞ is

a) $\frac{1}{25}$ b) $\frac{16}{25}$ c) $\frac{25}{16}$ d) $\frac{5}{4}$

Solution (c): $s_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{1}{1-\frac{1}{5}} + \frac{1 \times \frac{1}{5}}{\left(1-\frac{1}{5}\right)^2}$

$$= \frac{1}{\frac{4}{5}} + \frac{\frac{1}{5}}{\frac{16}{25}} = \frac{5}{4} + \frac{5}{16} = \frac{25}{16}$$

61) A ball is thrown vertically up in air where the resistive force can be considered to be constant. If time of ascent is t_1 and time of descent is t_2. The ratio of velocity of projection to the velocity just before it hits the ground is

a) $\frac{t_2}{t_1}$ b) $\left(\frac{t_2}{t_1}\right)^2$ c) $\frac{2t_2}{t_1}$ d) $\frac{t_2}{2t_1}$

Solution (a): The acceleration of ball during ascent is g+a. Where a is the acceleration due to resistive force. The time of accent, $t_1 = \frac{u}{g+a}$ (1)

The acceleration during descent g-a and time taken is $t_2 = \frac{v}{g-a}$ (2)

The time taken can also be written as

$$t_2 = \sqrt{\frac{2h}{g-a}} = \sqrt{\frac{u^2/g+a}{g-a}}$$

$$t_2^2 = \frac{u^2}{(g+a)(g-a)} \qquad (3)$$

Dividing equation (3) by equation (1) and (2)

$$\frac{t_2}{t_1} = \frac{u}{v}$$

62) Water level in a pool is 2 m. An iron pillar 3 m tall is placed in the water. If sun is 30° above the horizon, the shadow of the pole on the floor of the pool is

a) $2\sqrt{3}$ m.

b) Slightly less than $2\sqrt{3}$ m.

c) Slightly more than $2\sqrt{3}$ m.

d) $\sqrt{6}$ m.

Solution: (b) In absence of water the shadow of the tip of the pole would be at x = 2 tan 60 = $2\sqrt{3}$ m. Due to water the refracted ray bend towards the normal. Thus b is the right option.

63) The equations to the sides of a triangle are x – 3y = 0, 4x + 3y = 5 and 3x + y = 0. The line 3x – 4y = 0 passes through the

a) incentre b) centroid

c) circumference d) orthocenter of the triangle

Solution(d):Two sides 3x + y = 0 and x – 3y = 0 being perpendicular the given triangle is an right angled triangle hence (0, 0) is the orthocenter as the point of intersection of the lines is (0, 0). Hence the given line passes through the orthocenter

64) The area enclosed by 2|x| + 3|y| ≤ 6 is

a) 3 sq.units b) 4 sq.units c) 12 sq.units d) 24 sq.units

Solution(c): 2x + 3y ≤ 6 when x ≥ 0, y ≥ 0

2x – 3y ≤ 6 when x ≥ 0, y ≤ 0

– 2x + 3y ≤ 6 when x ≤ 0, y ≥ 0

– 2x – 3y ≤ 6 x ≤ 0, y ≤ 0

which represents a rhombus with sides

2x $\pm$ 3y = 6 and 2x $\pm$ 3y = $-$6

Length of the diagonals is 6 and 4 units along x-axis and y-axis

$\therefore$ The required area = $\frac{1}{2} \times 6 \times 4$ = 12 sq.units

65) The equation of the normal to the parabola $y^2 = 4x$ which makes an angle of 60° with x-axis is

a) $y = x\sqrt{3} - 5\sqrt{3}$ b) $y + x\sqrt{3} = 5\sqrt{3}$

c) $y = x\sqrt{3} + 5\sqrt{3}$ d) $y + x\sqrt{3} + 5\sqrt{3} = 0$

Solution(a): $y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

Slope of the normal is $\tan 60° = \sqrt{3}$

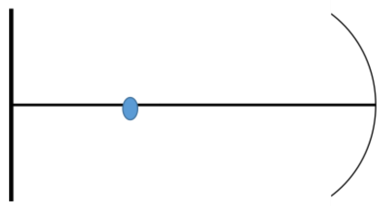
$\therefore$ Slope of the tangent is $-\frac{1}{\sqrt{3}}$

$\therefore \frac{2}{y} = -\frac{1}{\sqrt{3}}$ $\therefore -y = 2\sqrt{3}$ $\therefore y = -2\sqrt{3}$

$y^2 = 4x$ $\frac{4 \times 3}{4} = x$ $\therefore$ x = 3

$\therefore$ Equation of the normal is $y + 2\sqrt{3} = \sqrt{3}(x - 3)$

66) A plane mirror and a concave mirror of focal length 10 cm are 30 cm apart facing each other. A point object is placed 10 cm away from the plane mirror. Then,



a) number of images formed by the system is 2
b) number of images formed by the system is 3
c) number of images formed by the system is 1
d) distance between first two images formed is 20 cm.

Solution: (d) : Number of images formed is infinity. Image due to plane mirror is 10 cm behind the mirror while image due to concave mirror is at 20 cm in front of concave mirror. Thus the distance between first two images is 10 +10 = 20 cm

67) Three polaroids are kept coaxially. Angle between the first and third polaroid is 90°. Angle between the first and second polaroid is 60°. If light energy incident on the first polaroid is I_0. Light energy that emerges from the system is

a) zero b) $\frac{3I_0}{32}$ c) $\frac{3I_0}{16}$ d) $\frac{\sqrt{3}I_0}{8}$

Solution: (c): According to Malus law, $I = I_0 cos^2\theta$

After 2nd Polaroid, $I = I_0 cos^2 60 = \frac{I_0}{4}$

After 3 rd Polaroid, $I = \frac{I_0}{4} cos^2 30 = \frac{3I_0}{16}$

68) The value of $\lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{x^{10} + 10^{10}}$

a) 10 b) 100 c) 1000 d) 0

solution (a): $\lim_{x \to \infty} \frac{\left(1+\frac{1}{x}\right)^{10} + \left(1+\frac{2}{x}\right)^{10} + \ldots + \left(1+\frac{100}{x}\right)^{10}}{1+\left(\frac{10}{x}\right)^{10}}$ = 10

69) If tan θ + sec θ = p, then θ can be written as

a) $\sec^{-1}\left[\frac{1+p^2}{2p}\right]$

b) $\cos^{-1}\left[\frac{1+p^2}{2p}\right]$

c) $\tan^{-1}\left[\frac{2p}{2p^2-1}\right]$

d) $\tan^{-1}\left[\frac{p^2-1}{2p}\right]$

Solution (d) tan θ + sec θ = p

$(\tan\theta + \sec\theta)^2 = p^2 \Rightarrow \tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta = p^2$

$\tan^2\theta + 1 + \tan^2\theta + 2\tan\theta\sec\theta = p^2$

$2 + \tan^2\theta + 2\tan\theta\sec\theta = p^2 - 1$

$\tan\theta(\tan\theta + \sec\theta) = \frac{p^2-1}{2}$

$$\tan\theta = \frac{p^2-1}{2p} \Rightarrow \theta = \tan^{-1}\left(\frac{p^2-1}{2p}\right)$$

70) A boat carrying ten people is floating in a pond. Suppose all the ten people drink some water from the pond simultaneously then consider the two statements

(i) The fraction of the boat immersed in water increases marginally.

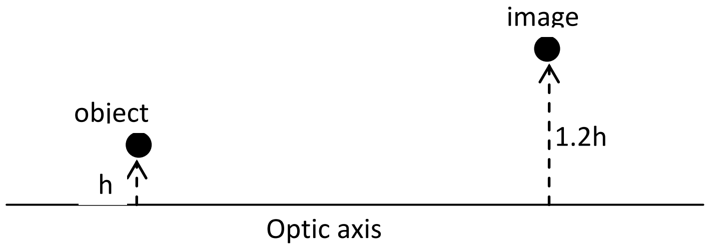
(ii) The level of the water in the pond will not change.

Choose the correct option

a) Statement (i) is correct while statement (ii) is incorrect
b) Statement (i) and (ii) are correct
c) Statement (i) is incorrect while statement (ii) is correct
d) Statement (i) and (ii) are incorrect

Solution (b): Statement one is correct since effective mass of boat has increased. Statement two is also correct since weight of water drunk by people in the boat will displace equal amount of water in the pond.

71) An optical system produces an image of an object as shown in the figure below. Guess the optical system and its position.



(i) concave mirror between the object and the image
(ii) biconvex lens on the left of the object.
(iii) convex mirror between the object and the image

a)(i) , (ii) and (iii) are possible
b) only (ii) and (iii) are possible
c)only (i) and (iii) are possible
(d) only (i) and (ii) are possible

Solution (d)

72) The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

a) 0
b) 2
c) 1
d) 3

Solution (a): $c_1 \to c_1 + c_2 + c_3$

$$\begin{vmatrix} 2\cos x + \sin x & \cos x & \cos x \\ 2\cos x + \sin x & \sin x & \cos x \\ 2\cos x + \sin x & \cos x & \sin x \end{vmatrix} = 0$$

$$(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

$$(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$(2 \cos x + \sin x) (\sin x - \cos x) = 0$

$2 \cos x = - \sin x$; $\quad$ $\sin x = \cos x$

$-2 = \tan x$ $\quad$ $\tan x = 1$

Which has no solution? $x = \frac{\pi}{4}$

$-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ $\therefore$ 1 solution

73) If tan A + sin A = m and tan A – sin A = n then $(m^2 - n^2)^2$ is

a) 15mn b) 16mn c) 18mn d) 4mn

Solution (b): tan A + sin A = m tan A – sin A = n

$(m^2 - n^2)^2 = [(\tan A + \sin A)^2 - (\tan A - \sin A)^2]^2$

$= (4 \tan A \sin A)^2 = 16 \tan^2 A \sin^2 A$

$= 16(\sec^2 A - 1) \sin^2 A = 16\left(\frac{1}{\cos^2 A} - 1\right)\sin^2 A$

$= 16(\tan^2 A - \sin^2 A) = 16\ mn$

74) ΔABC is a right angled triangle and C = 90° then tan A + tan B is

a) $\frac{b^2}{ac}$ b) a + b c) $\frac{a^2}{bc}$ d) $\frac{c^2}{ab}$

Solution (d): $\tan A = \frac{a}{b}$ $\tan B = \frac{b}{c}$

$$\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$$

75) The value of $\tan^{-1}\left(\frac{1}{a+b}\right) + \tan^{-1}\left(\frac{b}{a^2 + ab + 1}\right)$ is

a) $\tan^{-1} a$ b) $\cot^{-1} a$ c) $\tan^{-1} b$ d) $\cot^{-1} b$

Solution (b): $\tan^{-1}\left(\frac{1}{a+b}\right) + \tan^{-1}\left(\frac{a+b-a}{1+a(a+b)}\right)$

$\tan^{-1}\left(\frac{1}{a+b}\right) + \tan^{-1}(a+b) - \tan^{-1} a$

$\cot^{-1}(a+b) + \tan^{-1}(a+b) - \tan^{-1} a$

$\frac{\pi}{2} - \tan^{-1} a = \cot^{-1} a$

76) In a G.P of positive terms, for a fixed n, the n^{th} term is equal to sum of the next two terms. Then the common ratio of the G.P is

a) 2 cos 18° b) sin 18° c) cos 18° d) 2 sin 18°

Solution (d): $ar^n = ar^{n+1} + ar^{n+2} \Rightarrow r^2 + r - 1 = 0$

$$r = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$r > 0$ $\quad r = \dfrac{-1+\sqrt{5}}{2} = 2\sin 18°$

77) An equation of a circle touching the axes of co-ordinates and the line $x \cos\alpha + y \sin\alpha = 2$ is $x^2 + y^2 - 2gx + 2gy + g^2 = 0$ where g is

a) $2(\cos\alpha + \sin\alpha + 1)^{-1}$ b) $2(\cos\alpha - \sin\alpha + 1)^{-1}$

c) $2(\cos\alpha + \sin\alpha - 1)^{-1}$ d) $-2(\cos\alpha - \sin\alpha - 1)^{-1}$

Solution (b): Centre is $(+g, -g)$ radius is $|g|$
If it touches the lines $x \cos\alpha + y \sin\alpha = 2$,

$g \cos\alpha - g \sin\alpha - 2 = \pm g$

$g(\cos\alpha - \sin\alpha \pm 1) = 2$

$$g = \frac{2}{\cos\alpha - \sin\alpha \pm 1}$$

78) If A is a square matrix of order 3 and $|A| = 3$, then $|\text{adj } A|$ is

a) 3 b) 9 c) $\frac{1}{3}$ d) 0

Solution (b): If A is a square matrix of order n and $|A| \neq 0$ then $|\text{adj } A| = |A|^{n-1}$
$3^{3-1} = 3^2 = 9$

79) The number of solutions of $\sqrt{4-x} + \sqrt{x+9} = 5$ is

a) 0 b) 1 c) 2 d) 3

Solution (c)

$4 - x \geq 0$ and $x + 9 \geq 0 \Rightarrow -9 \leq x \leq 4$

$\sqrt{x+9} = 5 - \sqrt{4-x}$

$x + 9 = 25 - 10\sqrt{4-x} + 4 - x$

$10\sqrt{4-x} = 20 - 2x$

$\Rightarrow 5\sqrt{4-x} = 10 - x$

$25(4 - x) = 100 + x^2 - 20x$

$\Rightarrow x^2 + 5x = 0$

$x(x + 5) = 0$

$\therefore x = 0$ and $x = -5$

80) 2 ($\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$) is

a) 2π b) π c) $-\pi$ d) $\frac{\pi}{2}$

Solution (a): $\tan^{-1} 2 + \tan^{-1} 3 = \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right) + \pi$

$$= \tan^{-1}\left(\frac{5}{-5}\right) + \pi$$

$$= -\tan^{-1}(1) + \pi$$

$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

$\therefore$ Answer is 2π